

# Optimal Ground Motion Scaling under Spectral Variation Constraints

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## Abstract

In seismic design, scaling of ground motion records is critical for the accuracy of structural analysis. According to the Turkish Building Earthquake Code (TBEC 2018), for time history analyses, actual ground motions can be scaled according to the design spectrum. The selection of scale factors by the designer can distort the characteristics of the records and lead to erroneous structural analyses. Especially high scale factors can distort the characteristics of the original record. This study proposes a multi-criteria optimization process for determining scaling factors. The objective function was defined as minimizing the impact on the characteristics of the scaled acceleration records. Harmony Search Algorithm (HSA) and Grey Wolf Algorithm (GWO) were used in the optimization process. Additionally, the scaling process was carried out using the SeismoMatch program. The obtained results were compared in terms of the accelerograms and characteristics of the records. The effects of the optimum scale factors were also examined by conducting time-history analyses on a seven story reinforced concrete structure. According to the optimized results, the peak acceleration value in the accelerograms increased by 1.8 times, while scaling with SeismoMatch resulted in a 7.5 fold increase. Following an optimization process conducted on 11 different ground motion records, it has been demonstrated that the proposed variation-constrained method fully satisfies the TBEC 2018 criteria while exhibiting proficiency in preserving the seismological characteristics and energy content of the acceleration records.

## Keywords

ground motion scaling, optimization, hsa, gwo, TBEC 2018, spectral compatibility

## 1 Introduction

Time-history analyses produce the most realistic results among dynamic analyses performed for numerical models of structures. In time-history analyses, structural modeling is as important as the characteristics of the ground motions acting on the structure. Selecting the ground motions to be used and matching them with target spectra according to the structure's location is also a significant engineering problem. Many methods have been proposed for selecting acceleration records and adapting them to the design spectrum [1, 2].

In the early period, the effects of scaling elastic and elastic spectra on strength and displacement were investigated. In the studies conducted, variations such as Housner Intensity, Arias Intensity, Peak Ground Acceleration, and Peak Ground Velocity were examined by researchers [3]. Studies aiming for the similarity of selected acceleration records to the design spectrum have been carried out.

These similarities were achieved by considering acceleration, velocity, and soil parameters [4]. It has been suggested that the response spectra be modified with specific coefficients to make the records resemble the design spectrum [5]. In the proposed method, the coefficients used to modify the response spectra are determined according to distances. These studies, in general, are traditional approaches based on structural responses, characteristics of acceleration records, soil conditions, and distances.

With the increasing computational capacity of computers and the emergence of heuristic optimization algorithms, the usability of acceleration records in scaling methods has been proven. Heuristic optimization methods are very powerful tools for determining the optimum result under certain constraints. The selection of acceleration records and their approximation to design spectra is a complete optimization problem. Therefore, among the

first heuristic optimization algorithms, the Genetic algorithm was used by researchers for selection and scaling. In this study, the goal was to make the target spectrum and 7 scaled spectra similar to each other [6].

Scaling ground motions with genetic algorithms has been a pioneering study for researchers. Subsequently Jayaram et al. [7] developed a robust algorithm for matching both the mean and variance of a target response spectrum. This study proposes a computationally efficient, theoretically consistent ground motion selection algorithm to enable the selection of a set of ground motions with response spectra that have a target mean and target variance. They achieved this matching between target and sample means and variances by using Monte Carlo simulation and a greedy optimization technique, modifying the records one by one. Furthermore, Macedo and Castro [8] introduced the SelEQ framework, while Moschen et al. [9] and Mergos and Sextos [10] employed genetic algorithms to define the process within a multi-objective optimization context. The study selects and scales not only by achieving optimal matching with the target spectrum, but also by considering the seismology of the region, ground conditions, intensity and duration of strong ground motion, and the magnitude of scale factors. Kaveh et al. [11] performed comparative analyses on the efficiency of various multi-objective algorithms, and Kayhan et al. [12–14] proposed a solution model to obtain input ground motion datasets compatible with design spectra using the Harmony search algorithm. In their studies, they scaled ground motions that matched the design spectrum by selecting them from a large number of ground motions. Additionally, Georgioudakis et al. [15] established a multi-criteria framework that addresses both spectral fit and dispersion through evolutionary algorithms. Similarly, Georgioudakis et al. [16] utilized multi-criteria selection and scaling approaches to evaluate the seismic performance of steel frame structures under scaled ground motions. Wavelet based approaches proposed by Hancock et al. [17] and implemented in software such as SeismoMatch [18] remain among the most widely adopted tools in both research and engineering practice.

A significant portion of the proposed methods in the existing literature aim to approximate structural responses, soil conditions, and data record characteristics to the design spectrum. A review of these studies reveals that the goal is to optimize scaling coefficients by selecting from a large number of acceleration records. These approaches are difficult for users to implement due to the high number

of acceleration records they contain. Furthermore, uncontrolled high or low scaling coefficients can compromise the seismological and physical accuracy of acceleration records, jeopardizing the reliability of structural analysis results. Previous studies have shown that recording characteristics are distorted when the scaling factor is 4 or greater [19]. However, many studies have used scaling factors of 4 or greater in order to converge the target spectrum [20, 21]. Unlike previous studies, this work focuses directly on optimizing pre-determined records by the designer and emphasizes only scaling. The proposed method aims to minimize the variance between scaling coefficients, preventing them from taking the largest or smallest value in the optimization search space. This aims to ensure full compliance with the basic scaling rules in TBEC 2018, while minimizing the degradation of record characteristics [22].

To verify the accuracy and consistency of the results, the Harmony Search Algorithm (HSA) developed by Geem et al. [23] and the Grey Wolf Optimization (GWO) presented by Mirjalili et al. [24] were used. The performance of these two algorithms in solving the ground motion scaling problem was examined. In addition, the spectral fit capacity of the proposed method and its ability to preserve the characteristics of the acceleration series were comprehensively investigated by comparing the obtained data with the outputs from the SeismoMatch [18] software. The impact of the optimum scale factors was investigated by performing time history analyses on a seven story reinforced concrete building model.

## 2 Materials and methods

This study proposes a method using swarm-based optimization algorithms to match earthquake spectra to the target design spectrum. The target design response spectrum was created based on the Turkish Building Earthquake Code [22]. The scaling of the selected acceleration records was performed using Harmony Search (HSA) and Grey Wolf Optimization (GWO) algorithms. According to TBEC 2018, a summary of simple scaling, the proposed optimization based method, and its implementation are described in the following sections.

### 2.1 Simple scaling according to the Turkish Building Earthquake Code

The scaling of ground motion records in this study was conducted in strict accordance with the provisions of the Turkish Building Earthquake Code (TBEC 2018). The primary objective of this procedure is to ensure that the selected

earthquake acceleration time-series are representative of the seismic hazard level defined by the target design spectrum. According to TBEC 2018, simple scaling is performed to ensure the physical consistency and reliability of structural analyses by considering the following two criteria:

1. For each selected set of ground motions, the average of the 5% damped response spectra of the scaled recordings should not be lower than the target design spectrum (Fig. 1).
2. Scaling is performed over a period range consistent with the building's fundamental vibration periods. According to TBEC 2018, the average spectral acceleration of the scaled spectra should be greater than 1.3 times the amplitudes of the target spectrum, between  $0.2T_1$  and  $1.5T_2$ , where  $T_1$  represents the dominant period of the structure.

In Fig. 1,  $S_{D1}$  and  $S_{DS}$  show the spectral coefficients of the map including ground effects, while  $S_{ae}$  shows the spectral accelerations. These coefficients are taken from TBEC 2018 for the building location.

### 2.2 Proposed multi-criteria optimization approach

The proposed method is based on the Turkish Building Earthquake Code (TBEC 2018), considering simple scaling conditions for acceleration time series. In the simple scaling method, the response spectra of selected ground motions are magnified or reduced by multiplying them with a Scaling Factor (SF). According to TBEC 2018, at least 11 ground motion records should be used for time history analysis. As explained in the previous section, the average spectrum of the scaled records should not fall below 1.3 times the target design spectrum. The main challenge in this process is to determine the optimum scaling factors that give the closest average spectrum to the

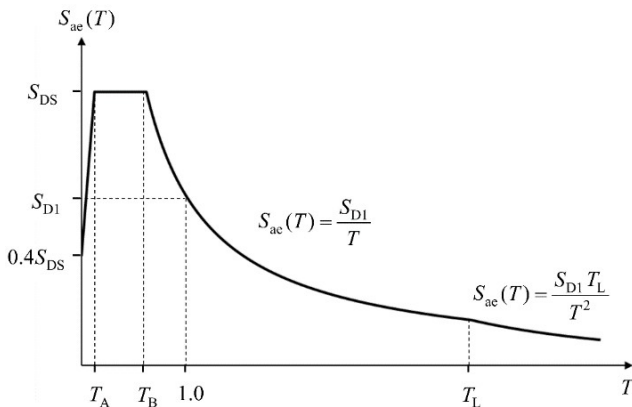


Fig. 1 Spectral acceleration-period graph according to TBEC [22]

target spectrum without compromising the natural characteristics of the ground motions. Consequently, this study focuses on finding the optimum values of these factors using two different objective functions:

1. Area Minimization: The first objective function aims to minimize the total area between the average of the scaled ground motion spectra and the target design spectrum. This function aims to ensure that the average of the obtained scaled records fits the target design spectrum with maximum accuracy. This objective function is given in Eq. (1):

$$Obj_1 = \begin{cases} \int_{0.2T_1}^{1.5T_1} |SA_{avg}(T) - SA_{target}(T)| dT, \\ SA_{avg} < SA_{limit} \\ \int_{0.2T_1}^{1.5T_1} |SA_{avg}(T) - SA_{target}(T)| dT \\ + Penalty_{TBEC}, SA_{avg} \geq SA_{limit} \end{cases}, \quad (1)$$

where,  $SA_{avg}$  represents the average of the scaled spectra, while  $SA_{target}$  symbolizes the target design spectrum. According to TBEC 2018, the mean spectrum of the scaled records must not fall below the target design spectrum within the specified period range. To strictly enforce this, a massive penalty value ( $10^8$ ) is added to the fitness if the mean spectrum violates this boundary at any period point (Eq. (2)):

$$Penalty_{TBEC} = 10^8 \cdot \sqrt{SA_{target} - SA_{avg}} \quad (2)$$

2. Multi-Criteria Variation Optimization: The second objective function incorporates the variation of the scale coefficients into the first objective function by multiplying it with a weighting factor. The primary goal of this function is to scale the original record with minimal change by moving the scale coefficients away from limit values. This prevents over-scaling of certain records, which could lead to a loss of physical realism. This hybrid objective function is presented in Eq. (3):

$$Obj_2 = \begin{cases} \int_{0.2T_1}^{1.5T_1} |SA_{avg}(T) - SA_{target}(T)| dT \\ + Penalty_{mean}, SA_{avg} < SA_{target} \\ \int_{0.2T_1}^{1.5T_1} |SA_{avg}(T) - SA_{target}(T)| dT \\ + Penalty_{mean} + Penalty_{TBEC}, \\ SA_{avg} \geq SA_{target} \end{cases} \quad (3)$$

To prevent the scaling factors (SF) from dispersing into unrealistic ranges and to keep the suite's intensity centered around a specific value, a weighting penalty is applied Eq. (4):

$$\text{Penalty}_{\text{mean}} = L_{\text{mean}} \cdot (\mu_{SF} - T_{\text{mean}})^2, \quad (4)$$

where  $\mu_{SF}$  is the mean of the current scaling factors. The weighting coefficient  $L_{\text{mean}}$  and the target mean scale factor  $T_{\text{mean}}$  used in the objective function were determined through a systematic calibration process involving multiple trial runs. This calibration was necessary to balance the trade-off between minimizing the spectral area difference and maintaining the physical authenticity of the ground motion records. It was observed that a value of  $L_{\text{mean}} = 25.0$ , combined with a  $T_{\text{mean}} = 1.8$  based on preliminary assessments of the unscaled records, provided the most stable convergence. This approach effectively guided the swarm-based search toward a solution that satisfies the target spectral thresholds while keeping the individual scaling factors within a statistically reliable and physically realistic range.

Fig. 2 presents a typical spectrum illustrating the minimized area within the objective function. The shaded region represents the spectral discrepancy between the target design spectrum and the scaled average spectrum, which the proposed optimization algorithms (HSA and GWO) aim to minimize.

### 2.3 Implementation of Harmony Search Algorithm (HSA)

In this study, the Harmony Search Algorithm (HSA), introduced to the literature by Geem et al. [23], is used as the basis for determining the optimal scaling factors, which

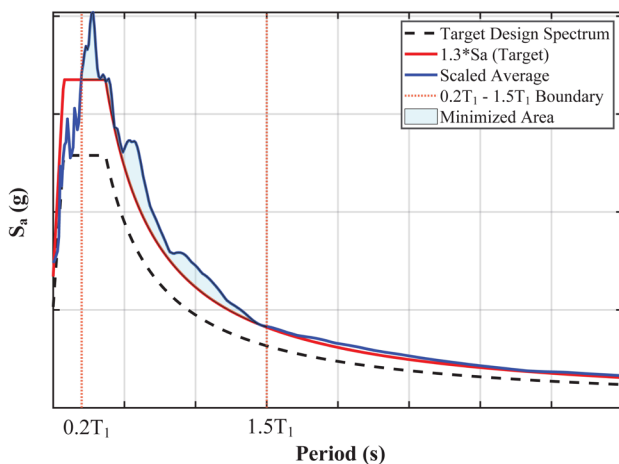


Fig. 2 Typical spectral acceleration plot representing the minimized area of the objective function

is a critical parameter in the scaling phase of earthquake ground motion records. HSA has been extensively applied to various engineering problems, consistently delivering robust performance [25–29]. Given its proven efficiency and superior convergence speed among existing meta-heuristics, this study utilizes HSA to achieve rapid and effective solutions. This algorithm is a heuristic optimization technique that models the improvisation process a musician uses to achieve the most harmonious sound. In the optimization model established in the research, each set of harmony represents the combination of scaling factors sought; the "perfect harmony" state represents the global optimum point where the defined objective functions reach their minimum value. The ability of HSA to explore the solution space and converge to the best value is based on the coordinated operation of control parameters defined as Harmony Memory (HM), Harmony Memory Consideration Ratio (HMCR), and Pitch Adjustment Ratio (PAR). Throughout the iterative process, the algorithm either re-evaluates the successful factors in the existing memory at each step or generates random new values to expand the search space. This dynamic mechanism ensures that the average scaled spectrum obtained remains in full compliance with the TBEC 2018 target spectrum, while also minimizing the variance between individual scaling factors. Thus, full compliance with regulatory standards is achieved, and the consistency of the analytical model is maintained by minimizing discrepancies between records.

### 2.4 Implementation of Grey Wolf Optimizer (GWO)

In this study, another algorithm applied in the process of determining the most ideal scaling factors for the selected 11 ground motion records is the Gray Wolf Optimization (GWO) algorithm proposed by Mirjalili et al. [24]. GWO was selected for this study due to the accurate results it provides within swarm-based search methodologies, particularly in addressing complex civil engineering problems [30–33]. The method is based on the leadership hierarchy and hunting strategies of gray wolves. In this system, the population is divided into four different authority levels: Alpha, Beta, Delta, and Omega. The most ideal scaling coefficient set in the current iteration of the optimization is positioned as the Alpha leader, while the second and third most qualified solution sets following it are positioned as Beta and Delta, respectively. The basic strategy of the algorithm is to leave the authority to predict the location of the target point (optimal solution) in the search space entirely to these first three-level

leadership cadres. The Omega-type candidates, which constitute the rest of the system, update their positions based on the coordinate data from the leader solutions, instead of independently determining a direction on their own. This hierarchical dependency ensures that the algorithm focuses in an organized and concentrated manner on the regions with the highest potential, rather than moving randomly across the search space. In this study, Alpha was assigned as the scaling factor set that yielded the closest result to the TBEC 2018 target spectrum. During the implementation phase, GWO not only minimized spectral deviation but also minimized the disruptive effects on the energy content and seismological characteristics of the records, taking into account the variation constraints proposed in the study. This optimization process, performed on 11 records, confirms that GWO offers high convergence stability and low computational cost in precision-requiring engineering problems such as seismic scaling.

### 3 Numerical application

In this section, a comprehensive numerical application is conducted to demonstrate the effectiveness of the proposed multi-criteria optimization framework in seismic scaling procedures. The implementation follows a systematic procedure involving the definition of the target spectrum, the establishment of database selection criteria, and the configuration of the algorithmic parameters.

#### 3.1 Selection of target design spectrum

In this study, scaling was performed for target design spectra in accordance with TBEC 2018 [22]. The target design spectrum represents the seismic hazard of the location of the structure. The target design spectrum in accordance with TBEC 2018 was defined by obtaining it from the AFAD [34]. The coordinates of the state hospital located in Ceyhan district of Adana province were selected. For this specific location, the map spectral acceleration coefficients were determined as  $S_s = 1.273$  and  $S_1 = 0.322$ . The DD1 earthquake ground motion level, representing the highest seismic demand, was selected, and Site Class ZC was determined to reflect the local soil conditions. The DD1 ground motion level represents the largest earthquake effect considered in the design code, characterized by a 2% probability of exceedance within a 50 year period, which corresponds to a return period of 2475 years as defined in TBEC 2018. In addition, a damping ratio of 5% was used for the spectra. The relationship between the applied target spectrum and these period limits is shown in Fig. 3.

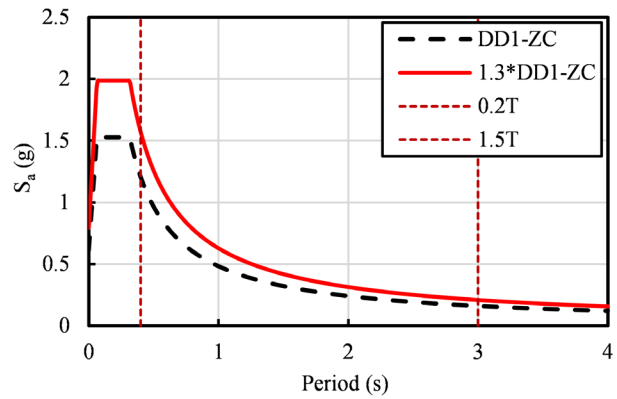


Fig. 3 DD1-ZC design spectrum and optimization boundaries for the selected location [22]

#### 3.2 Selection of ground motion records

The acceleration records used in the scaling process were meticulously selected from the PEER NGA-West2 database to ensure compatibility with the seismic hazard analysis [35]. Three primary selection criteria were applied during the database search: a moment magnitude ( $M_w$ ) between 6.0 and 8.0 to represent appropriate seismic energy release, an average shear wave velocity in the top 30 meters ( $V_{s30}$ ) ranging from 360 m/s to 760 m/s for compatibility with the site class, and a maximum Joyner-Boore distance ( $R_{jb}$ ) of 30 km to account for near-fault effects. A total of 11 ground motion records satisfying these criteria were identified. The seismological characteristics and station information of the selected records are presented in Table 1.

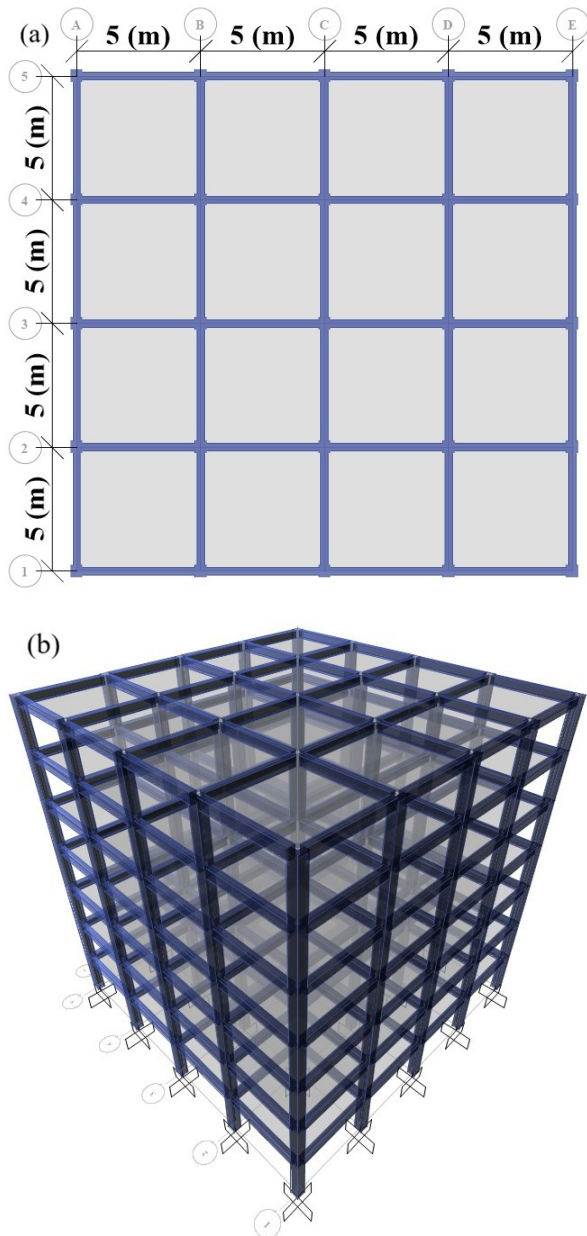
#### 3.3 Test building

A representative building was generated to investigate the impact of optimized scaling factors on structural analyses. The structural responses were evaluated by conducting time-history analyses using scaled acceleration series obtained from Function 1, Function 2, and SeismoMatch. For this purpose, a seven-story, three-dimensional reinforced concrete (RC) building was selected as the model. The test structure is symmetrical in plan, with equal 5 meters in all spans and story heights of 3 meters on each floor (Fig. 4).

Structural components such as columns, beams, and slabs are designed in compliance with TBEC 2018 and TS500 standards [22, 36]. The material properties are defined with a compressive strength of 30 MPa for concrete, and yield/tensile strengths of 420 MPa and 500 MPa for steel, respectively. Frame elements are utilized to model columns and beams, while a rigid diaphragm is assigned to each story. The cross-sections are specified as  $500 \times 500$  mm for columns,  $500 \times 300$  mm for beams, and a thickness of 150 mm for slabs. Gravity loads include a

**Table 1** Characteristic properties of the selected ground motion records [35]

RSN	Earthquake	Location	Year	$M_w$	Component	PGA (g)	$R_{jb}$ (km)	$V_{s30}$ (m/s)
451	Morgan Hill	Coyote Lake Dam - Southwest Abutment	1984	6.19	CYC195	0.927	0.53	561.43
1085	Northridge-01	Sylmar - Converter Sta East	1994	6.69	SCE281	0.662	5.19	370.52
1510	Chi-Chi, Taiwan	TCU075	1999	7.62	TCU075-E	0.713	0.89	573.02
3746	Cape Mendocino	Centerville Beach, Naval Fac	1992	7.01	CBF360	0.449	18.31	459.04
4040	Bam, Iran	Bam	2003	6.60	BAM-T	0.332	1.7	487.4
4097	Parkfield-02, CA	Slack Canyon	2004	6.00	SCN360	0.478	2.99	648.09
4451	Montenegro, Yugoslavia	Bar-Skupstina Opstine	1979	7.10	BSO000	0.629	6.98	462.23
4482	L'Aquila, Italy	L'Aquila - V. Aterno -F. Aterno	2009	6.30	CU104YLN	0.349	6.55	552
4847	Chuetsu-oki, Japan	Joetsu Kakizakiku Kakizaki	2007	6.80	65010EW	0.372	11.94	383.43
6928	Darfield, New Zealand	LPCC	2010	7.00	LPCCS10E	0.440	25.67	649.67
8164	Duzce, Turkey	IRIGM 487	1999	7.14	487-NS	0.456	2.65	690



**Fig. 4** Test building views: (a) Plan and (b) 3D view

uniform dead load of  $10 \text{ kN/m}^2$  and a live load of  $2 \text{ kN/m}^2$  on the slabs, supplemented by a  $6 \text{ kN/m}$  wall load on the beams. Following TS500, a load combination of  $G + 0.3Q$  (excluding self-weight) is implemented to define the mass source for time-history analyses. The fundamental vibration period of the structure was determined to be  $2.0 \text{ s}$ . The entire structural system is assumed to exhibit linear elastic behavior throughout the analyses. Furthermore, due to the symmetrical plan geometry of the building, seismic analyses were performed in a single principal direction. The numerical simulations are performed using the ETABS v22 software, a widely recognized finite element platform for reinforced concrete modeling [37].

### 3.4 Optimization parameters and configuration

This section presents the parameters used in the optimization process for the numerical example. Firstly, the search space representing the scaling coefficient was chosen as  $0.5\text{-}4$ , as in many previous studies. This constraint ensures that ground motion records remain realistic from an engineering perspective by preventing them from being reduced to very low values or increased to excessively high amplitudes that could lead to numerical instabilities in structural analyses. Furthermore, in accordance with TBEC 2018, the fundamental period of the structure defining the scaling range was selected as  $T_1 = 2.0$  seconds. For  $T_1 = 2.0$ , the scaling range was calculated to be between  $0.4$  and  $3.0$  seconds.

The optimization process in this study was established as a robust benchmark on a flexible structural model with a natural vibration period of  $2$  seconds to specifically test the proposed algorithm's performance and sensitivity to spectral variations in the long-period range. Similarly, the selection of the DD1 earthquake level aims to demonstrate the method's effectiveness under extreme seismic conditions where scaling factors typically reach their limit

values. It is anticipated that the proposed method would converge more easily and yield even more stable results for more frequent earthquake levels (e.g., DD2), where target spectral demands are lower. Consequently, validating the success of the methodology under the most demanding scenario of DD1 and  $T = 2s$  inherently supports its applicability to less intense seismic levels and various structural configurations. Furthermore the method can be easily applied to different soil classes. However, ground motions with characteristics ( $V_{S_{30}}$ ) suitable for the varying soil classes must be selected.

Additionally, the specific search parameters of the optimization algorithms are presented. These parameters, appropriately determined for the problem based on preliminary studies, are given in Table 2.

As presented in Table 2, to compare the performance of the algorithms, both optimization processes were performed using the same population size and iteration capacity. The Harmony Memory Size (HMS) of HSA and the Pack Size of GWO were set to 50. In addition, to ensure that both methods could converge to the global optimum, the analyses were performed on a wide solution range of 100000 iterations. Furthermore, the optimization

process was performed using the Matlab R2024b environment [38]. Computational analyses were performed on a system equipped with an 11th Gen Intel Core i5-1135G7 (2.40 GHz) processor and 8.00 GB RAM.

#### 4 Result and discussion

In this section, the seismic ground motion scaling performance of the developed meta-heuristic optimization framework is comprehensively analyzed. The evaluations were carried out in three main stages: first, the convergence speeds and computational efficiencies of the algorithms were examined; second, the compatibility of the resulting scaled spectra was tested against regulatory criteria and traditional methods; and finally, the effects of the scaling process on the seismological characteristics of the ground motions were detailed.

The optimal scaling factors calculated for each of the 11 ground motion records are given in Table 3. The results are categorized based on the applied optimization algorithms (HSA and GWO) and the specific objective functions used during the analysis. Column 'Func. 1' represents the area-focused optimization, while 'Func. 2' illustrates the results where the multi-criteria variation constraint was incorporated.

The optimized scaling factors obtained for the 11 selected ground motion records are presented in Table 3. The optimum scaling factors were successfully optimized within the limit values for all functions and ground motions. HSA and GWO calculated similar scaling factors in both objective functions. These obtained scaling factors indicate that the two algorithms are mutually reinforcing.

The results obtained by area oriented optimization (Function 1) showed scaling factors reaching both the

**Table 2** Configuration of optimization algorithms

Algorithm	Parameter	Value
HSA	Harmony Memory Size (HMS)	50
	Harmony Memory Considering Rate (HMCR)	0.90
	Pitch Adjusting Rate (PAR)	0.35
	Maximum Iterations	100000
	Number of Wolves (Pack Size)	50
GWO	Convergence Constant (a)	2 → 0
	Maximum Iterations	100000

**Table 3** Optimized scaling factors (SF) obtained by HSA and GWO algorithms for two different objective functions

Ground Motion	HSA		GWO	
	Area (Func. 1)	Area + Variation (Func. 2)	Area (Func. 1)	Area + Variation (Func. 2)
RSN451	1.556300681	1.880388854	1.841518309	1.881800401
RSN1085	0.541282685	1.866239914	0.601882229	1.864989097
RSN1510	2.74736386	1.866203948	3.153158336	1.866797459
RSN3746	2.530308831	1.867687251	0.500132028	1.869313801
RSN4040	0.746059974	1.865078326	1.124436609	1.865096555
RSN4097	0.507030283	1.861360402	0.513275262	1.861008024
RSN4451	0.500000000	1.861970174	1.000308975	1.860067877
RSN4482	3.709802233	1.879357413	3.895855309	1.878080604
RSN4847	0.610381338	1.862061964	0.572700635	1.861963915
RSN6928	2.403115853	1.857272563	2.599822401	1.855417129
RSN8164	3.906502299	1.866096606	3.991473121	1.866369459

lower and upper limit values. In particular, the RSN8164 record reached the upper limits in both HSA and GWO. Although the scaling coefficient of this record remained within the limit values, its characteristics showed a relatively large change compared to the original record. Furthermore, broadening the scaling factor constraints may lead to a more effective minimization of the objective function. Nevertheless, excessive scaling tends to diminish the physical authenticity of the acceleration records and leads to a higher degree of variance among them.

When the results of area+variation oriented optimization (Function 2) are examined, it is seen that the scaling coefficients converge to 1.86 for all records. The closeness of the scaling factors obtained in Function 2 is consistent with the proposed objective function. None of the scaling factors found by the proposed function 2 converged to the limit values; minimum scaling coefficients satisfying the TBEC 2018 requirements were determined. This minimized the degradation of the originality of all scaled records. It is assessed that the accuracy of structural dynamic analyses performed with the obtained optimum scaling factors will increase.

#### 4.1 Convergence and computational performance analysis of optimization algorithms

In this study, which aims to optimize scaling factors, two different swarm-based optimization algorithms were utilized to cross-validate the results. Furthermore, the solution search processes and speeds of the HSA and GWO algorithms were examined to evaluate the numerical efficiency of the methods. Fig. 5 illustrates the convergence characteristics exhibited by both algorithms under different objective functions over 100000 iterations.

Fig. 5 illustrates the convergence curves of both algorithms over 100000 iterations. In the area-focused objective function (Fig. 5(a)), it is observed that the GWO algorithm reaches lower cost levels much faster than the HSA and converges toward the optimum in the earlier stages of the iterations. In the multi-criteria objective function (Fig. 5(b)), although the HSA algorithm starts with higher initial cost values, it exhibits a sharp decline after

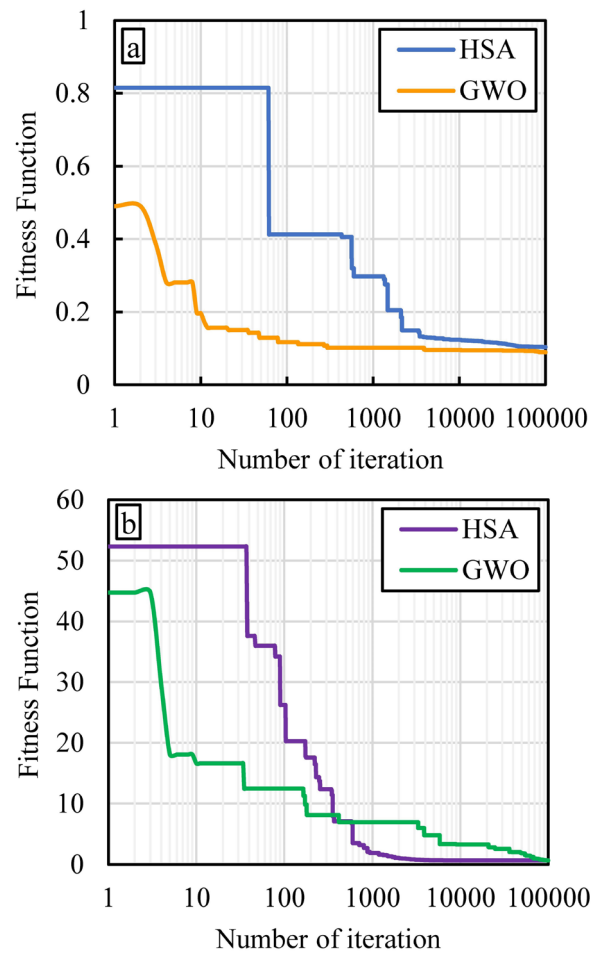


Fig. 5 Convergence history of the optimization algorithms: (a) Area-focused objective function (Function 1), (b) Multi-criteria objective function targeting area and variation (Function 2)

approximately 1000 iterations, eventually reaching a final level that is as stable as, or even more consistent than, the GWO. The stable plateaus achieved by both algorithms at the end of the 100000 iterations in both graphs confirm that the algorithms successfully reached the global minimum and that the solution is robust. The final statistical data, computational durations, and spectral compatibility performances are comparatively presented in Table 4.

Table 4 shows the objective function values and computational costs of the proposed functions. Examining the objective function value, it is seen that GWO produces slightly better values for both objective functions.

Table 4 Comparison of optimization algorithms and objective functions based on performance criteria

Algorithm/Function	Objective function value	Area	Variation	Time (s)
HSA/Area (Function 1)	0.1037769	0.103415	1.3195	1.69
HSA/Area+Variation (Function 2)	0.6612100	0.544849	0.0072	1.79
GWO/Area (Function 1)	0.0894012	0.089395	1.3802	44.08
GWO/Area+Variation (Function 2)	0.6610000	0.544672	0.0077	51.12

The objective function value in Function 2 is higher than in Function 1, as expected, considering the variation in scaling factors. Similarly, the calculated area is also higher in Function 2 compared to Function 1. However, the variation between scaling factors is significantly reduced in Function 2. This result shows that both algorithms can find the global optimum by reaching similar fitness values; however, Function 2 stands out in terms of preserving the original characteristics of seismic records. The variation values calculated by HSA are lower than those of GWO. This indicates that the scaling factors are calculated to be more similar to each other in HSA. Furthermore, when the computational costs are examined, it is found that HSA finds results much faster than GWO. Thus, it shows that HSA can be used effectively in more complex problems.

#### 4.2 Comparisons of scaled spectra

Following the algorithmic performance analysis, the compatibility levels of the optimized ground motion records with the target design spectrum were evaluated. In this context, both the effects of different objective functions and the success of the developed methods were examined through spectral plots.

The final compatibility of the scaled ground motions obtained through the proposed optimization processes with the TBEC 2018 design spectrum, as well as the distribution characteristics of the scaling factors, are visualized in Fig. 6.

Following the comprehensive numerical data and statistical comparisons, the final compatibility of the optimized ground motion records with the target spectrum is visualized in Fig. 6. Fig. 6 presents the scaled spectra produced by both algorithms under different objective functions, along with their mean (green curve), in comparison with the design spectrum.

As shown in all plots, the scaled mean spectral acceleration, represented by the green curve, strictly satisfies the TBEC 2018 criteria by not falling below the target design spectrum ( $1.3 \cdot DD1-ZC$ ) within the required period range of  $0.2T_1$  to  $1.5T_1$  (indicated between the red dashed lines). The area between the mean spectral acceleration obtained by the proposed area-and-variation constrained method (Function 2) and the  $1.3 \cdot DD1-ZC$  limit is larger compared to the area-only method. Although the spectral acceleration amplitudes slightly increase in the proposed method, the proximity of the acceleration series to each other and to their original characteristics is considered an acceptable engineering trade-off. Consistent with the low standard deviation data in Table 4, the more homogenous clustering of records around the mean in the Func 2 results (plots b and d) proves that the proposed method generates more reliable data sets for seismic analysis. To evaluate the performance of the proposed optimization methods against traditional approaches, the results obtained using the widely recognized SeismoMatch software are presented in Fig. 7.

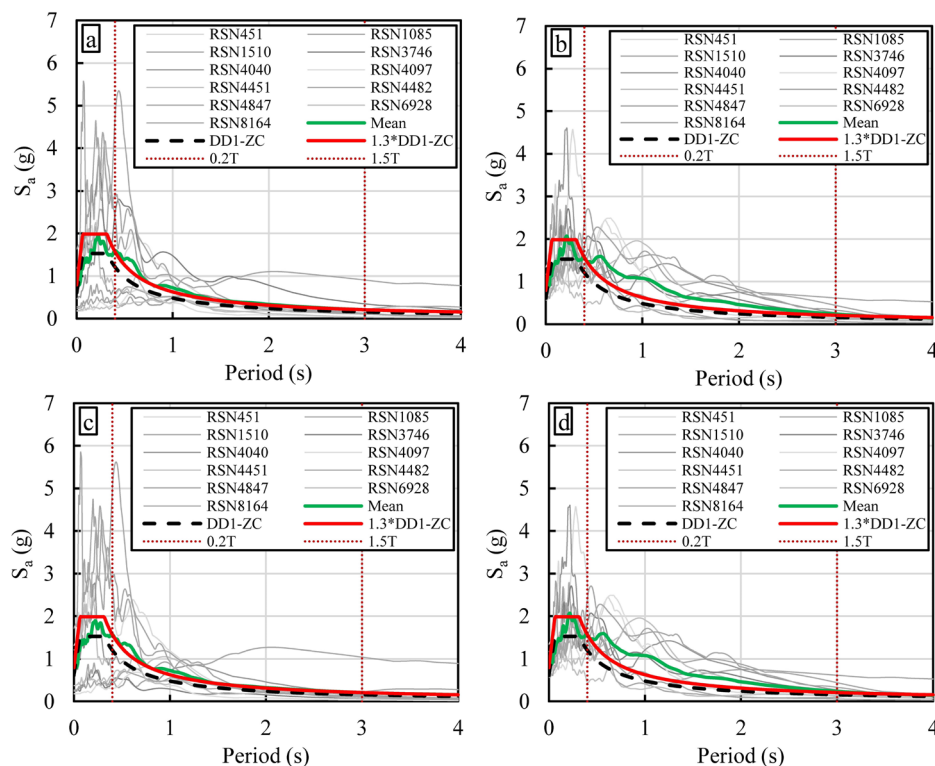


Fig. 6 Scaled spectral acceleration-period plots: a) HSA Func 1, b) HSA Func 2, c) GWO Func 1, d) GWO Func 2

When Fig. 7 is examined, it is observed that while the scaled mean spectrum (green curve) produced by SeismoMatch is compatible with the target spectrum, it exhibits higher peak values in spectral acceleration amplitudes compared to our optimization-based methods. The proposed multi-criteria optimization method (Func. 2) provides a more controlled scaling compared to the SeismoMatch results, better preserving the original earthquake characteristics of the records and ensuring a statistically more homogeneous dataset. To demonstrate the performance of the proposed method, a direct comparison of the mean spectra obtained by both algorithms with the outputs of the SeismoMatch software is presented in Fig. 8.

When Fig. 8 is examined, it is observed that the mean spectrum produced by SeismoMatch (light blue curve) exhibits excessive values by staying significantly above the target spectrum, particularly in certain period ranges. In contrast, the mean spectra obtained through the

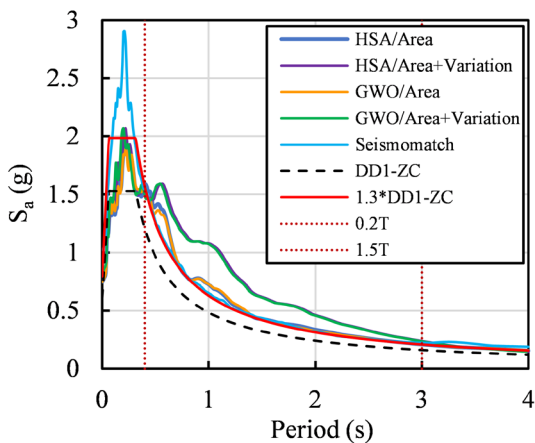


Fig. 7 Spectral scaling results obtained using SeismoMatch software

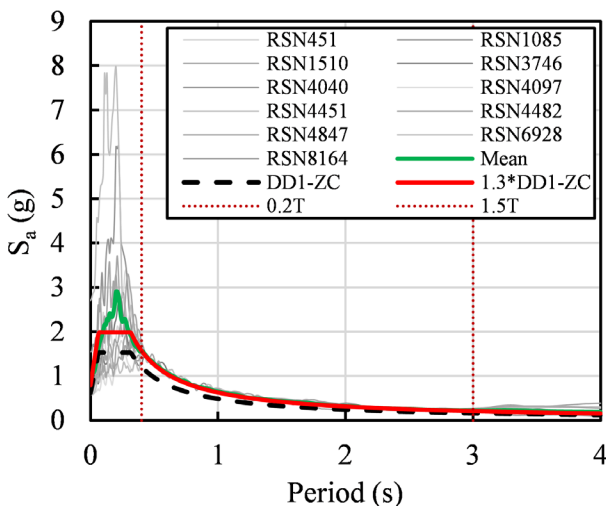


Fig. 8 Comparison of the mean spectra obtained by the proposed optimization methods and SeismoMatch

proposed multi-criteria approach remain just above the target spectrum, demonstrating a much more balanced and economical fit. This confirms that the developed optimization methods succeed in keeping the mean spectral acceleration levels as close as possible to the target spectrum, thereby minimizing the over-scaling error often observed in traditional software.

### 4.3 Evaluation of accelerograms and seismological characteristics

In this section, the effects of the scaling factors obtained through the proposed methods on the accelerogram structures and seismological characteristics of the ground motions are evaluated. Among the 11 ground motion records used in the analyses, the Darfield (RSN 6928) record, which reflects the general trend and demonstrates significant compatibility, was selected for detailed examination. Fig. 9 illustrates the original acceleration data for RSN 6928, the predefined scaling limits (0.5, 4.0), and the scaled acceleration-time histories produced by the GWO/area (Function 1), GWO/area + variation (Function 2) methods, and the SeismoMatch. Since the HSA and GWO algorithms yielded nearly identical optimum scaling factors, only the results obtained via the GWO algorithm are presented in Fig. 9 to ensure visual clarity.

Fig. 9 clearly shows the differences in scaled acceleration time series and peak amplitude (PGA) values. The peak acceleration values for the original record (a), GWO/Area (Function 1) (d), GWO/Area+Variation (Function 2) (e), and SeismoMatch (f) were calculated as 0.357 g, 0.927 g, 0.662 g, and 2.704 g, respectively. Furthermore, acceleration-time graphs obtained with limiting scaling coefficients for comparison with optimum results are given in Figs. 9(b) and 9(c).

According to the accelerograms in Fig. 9, the scale coefficients were calculated as 2.6 and 1.86 by Function 1, which minimizes the field difference, and Function 2, which minimizes both field difference and variation. With this difference in scale coefficients, the peak amplitude values in the accelerograms were calculated as 0.927 g and 0.662 g. With the proposed Function 2, the peak amplitude was reduced by approximately 30%. This reduction will certainly increase in records with high scale coefficients, such as RSN8164. The fact that SeismoMatch (Fig. 9(f)) significantly alters the peak amplitude, increasing it to approximately 7.5 times the original value, increases the risk of distorting the physical nature of ground motion with traditional software. The scaled accelerograms calculated by SeismoMatch suggest approximately twice



Time-history analyses were conducted using both the original and scaled acceleration-time series. Due to the similarity between the optimum scaling factors computed for Functions 1 and 2 using HSA and GWO, the scaling factors determined by GWO were employed. Fig. 10 illustrates the mean story accelerations obtained from the eleven scaled ground motion records.

Upon examining the analysis results for the seven-story building model in Fig. 10, it is observed that acceleration values increase consistently with story height for all scaling methods, reaching their peak at the seventh story. While the 4\*Original series generates the highest acceleration demands at approximately 25 m/s<sup>2</sup>, the GWO/Area method produces significantly lower acceleration values across all levels, exhibiting a more stable profile compared to other approaches. Notably, while the GWO/Area + Variation method shows a similar overall trend to SeismoMatch, it yields slightly higher story accelerations than the series calculated by SeismoMatch across all floors, a discrepancy that becomes more pronounced at the upper story levels.

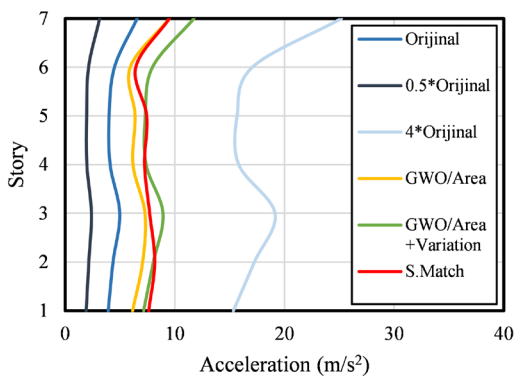
As anticipated, this result is consistent with the spectral acceleration increases observed in the acceleration-time spectra, stemming from the fact that the proposed GWO/Area + Variation method utilizes lower scaling factors compared to alternative methods. The use of minimized scaling factors ensures that the inherent characteristics of the ground motion records are preserved while remaining parallel to the spectral demand increases reflected in the story acceleration responses. While this increase remains relatively minor, it ensures that the structural analyses stay on the conservative side of the design. Fig. 11 illustrates the peak story accelerations derived from the original and scaled acceleration-time series, highlighting the maximum responses across all story levels.

Fig. 11 presents the results of the acceleration record that

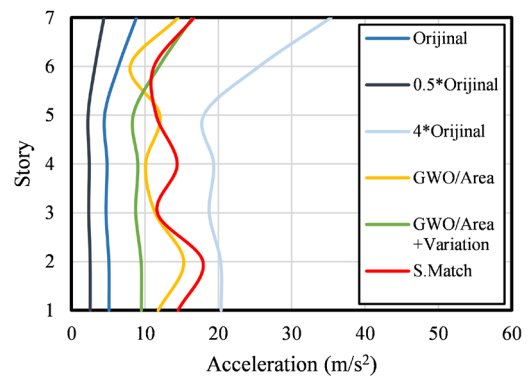
yielded the maximum story acceleration responses among the analyses conducted using different scaling methods in Fig. 11. The story acceleration values calculated using the scaling factors obtained via GWO/Area + Variation were generally found to be lower than those of SeismoMatch and GWO/Area. Since the proposed method multiplies all acceleration records by an optimum scaling factor, the peak acceleration demands could be calculated at lower, more optimized levels. In contrast, other methods resulted in higher story accelerations due to the utilization of high scaling factors that often reach limit values. These results reveal that not only the mean of the acceleration records used, but also the critical acceleration records that calculate the maximum story accelerations should be carefully examined.

### 5 Conclusions

In this study, real ground motions were scaled using swarm-based optimization algorithms. Scaling was performed using a simple scaling method, considering the principles of TBEC 2018. Two different objective functions were used to optimize the scale coefficients. The first function aimed only at convergence to the target spectrum, while the second function aimed to minimize the variation of the scale coefficients in addition to the first function. This minimized the difference between the original and scaled records in terms of seismological characteristics and energy parameters. The Harmony Search Algorithm (HSA) and the Grey Wolf Algorithm (GWO) were used in the optimization process. Additionally, ground motions were scaled using SeismoMatch, a wave-based scaling tool widely used by designers. The effectiveness of the proposed method was evaluated by comparing spectral compatibility, computational efficiency, and seismological characteristics. The results show that the optimization processes were successfully completed using swarm-based algorithms.



**Fig. 10** The average story accelerations derived from the eleven original and scaled acceleration-time series, using various scaling methods



**Fig. 11** Maximum story accelerations obtained from the most critical scaled acceleration records

Optimal scale coefficients were determined in full compliance with the TBEC 2018 constraints.

It was determined that the optimum scale coefficients obtained only with the objective function (Function 1) that minimizes the field difference reached limit values (0.5 or 4). High scale coefficients also resulted in high amplitude values in acceleration-time series. The inclusion of variation constraint (Function 2) in field minimization moved the scale coefficients away from limit values. However, it was observed that the convergence of the average spectrum to the target spectrum decreased compared to function 1. As a result, although the scaled average spectrum deviated slightly from the target spectrum, records closer to the original records were obtained. When the scaling results obtained with SeismoMatch are examined, the acceleration record characteristics change much more in SeismoMatch compared to the proposed optimization-based methods. For the Darfield record, SeismoMatch increased the PGA by 7.5 times, while the

proposed method (Function 2) kept this increase at a physically realistic level of 1.8 times.

When the performance of GWO and HSA was examined, although similar results were obtained, it was determined that GWO's objective function value was slightly superior. In the computational cost comparison, HSA, GWO, and SeismoMatch completed the scaling process in 1.79, 51.12, and 282 seconds, respectively. This shows that HSA is considerably faster than the other methods.

Considering all the results, unlike the highly over-scaled results frequently encountered in previous studies, the proposed variation-constrained method offers a fast approach that is both fully compliant with TBEC 2018 and minimizes the distortion of the original ground motion characteristics. For future research, developing a user-friendly interface (GUI) to facilitate the widespread use of the proposed optimization framework by engineers and researchers is a priority goal.

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