REGULATION MODEL OF THE LAKE VELENCE RESERVOIR SYSTEM

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Introduction

A Hungarian water resources system consisting of *Lake Velence* and two upstream reservoirs will be analyzed from the aspect of meeting optimum recreation conditions in the lake region.

The integrate, complex development program of the Lake Velence recreation area requires the lake level to be regulated to meet recreation demands. Regulation will be achieved by the *Zámoly* reservoir, formerly completed, and by the recently constructed *Pátka* reservoir. The two reservoirs and the outlet gate of Lake Velence will maintain the optimum lake level over extended periods of time.

A model has been established for estimating damages concomitant to lake levels other than ideal (high or low) to find the optimum schedule of operation, so as to minimize the average value of the damages resulting from level fluctuations, rather than the fluctuations themselves. In what follows the mathematical model and its practical application will be described.

The water resources system has to be operated in compliance with objectives set out before. During the past two or three decades, important advances have been made in the mathematical modelling of water resources systems [1, 2, 3, 4, 5, 6]. These have proved successful, among others, in optimizing operating schedules.

Description of the system

Lake Velence, with an area of 26 sq·km, is situated in the western part of Hungary, not very distant from *Lake Balaton*. In its catchment area of 615 sq·km two reservoirs have been constructed, namely the reservoirs *Zámoly* and *Pátka*, with volumes of $4,5 \cdot 10^{\cdot}$ and $8 \cdot 10^2$ cu·m, respectively *(Fig. 1)*.

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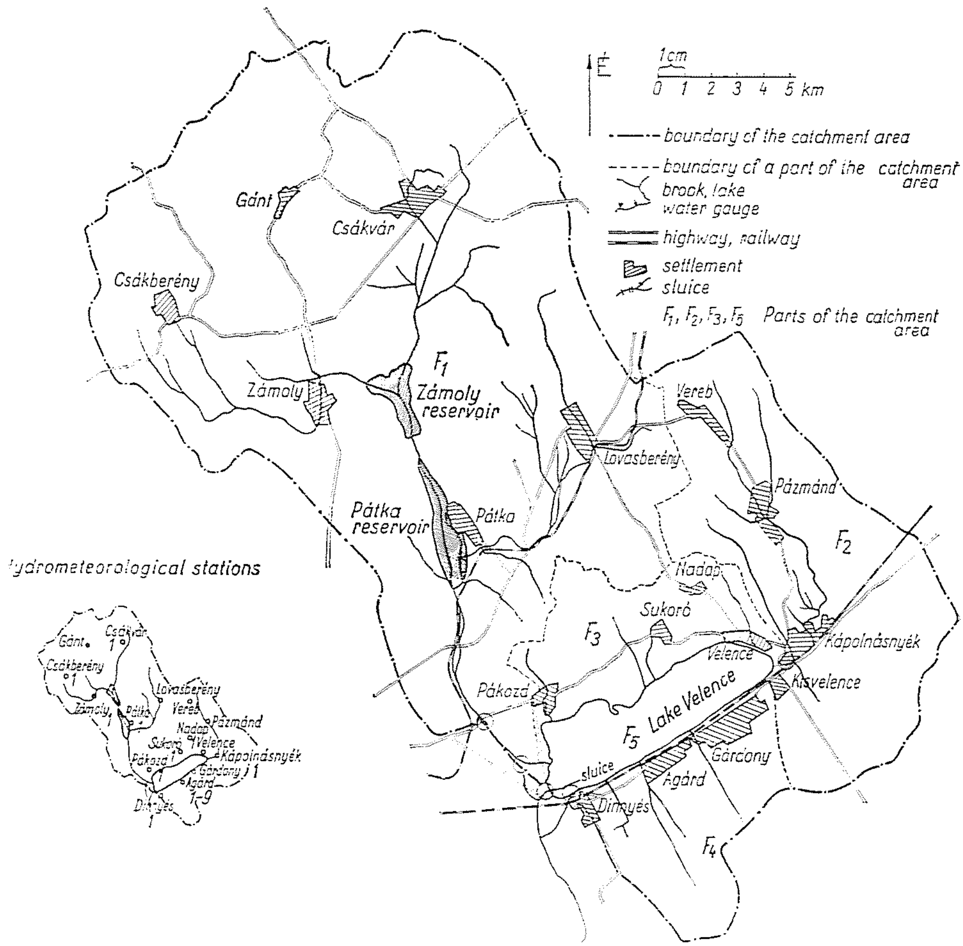

Fig. 1. The catchment area of Lake Velence. Parts of the catchment area: catchment area of the brook Császárvíz......400 km^2; catchment area of the brook Vereb-Pázmánd.....123 km^2; catchment area of the northern shore of Lake Velence 43 km^2; catchment area of the southern shore of Lake Velence ...49 km^2; area of Lake Velence at medium water level ....255 km^2

The area around Lake Velence is appreciated for its recreational value in Hungary. Recreation aspects require to maintain the lake level constant within narrow limits. Deviation from this optimum level is likely to cause damages, either direct ones (inundation, damages to buildings at high levels, or biological, water quality deterioration at low levels) or indirect ones (inferior recreation conditions). Since under natural conditions the lake level fluctuated widely around the ideal value, artificial regulation has been considered. The two reservoirs at Zámoly and Pátka have been constructed for this purpose. For operating the gates in the reservoirs, and at the outlet from Lake Velence, suitable instructions had to be elaborated.

The water released from the series-connected reservoirs is conveyed into the recipient Lake Velence by a brook called *Császárvíz*. The excess water from the lake is released through the gate at *Dinnyés*, capable of discharging up to 4 cu · m/sec. 60 per cent of the lake surface is covered by reed. Mean value of the precipitation on the lake surface is $P = 600$ mm, the inflow $I = 900$ mm, evaporation from the lake $E = 950$ mm and outflow amounts to $O = 550$ mm. The mean water budget is balanced:

$$P + I = E + O \tag{1}$$

but shortages are frequent in the catchment. An extensive network of precipitation gauges has been established (with records back to 1900), discharge measurements started in 1952 at *Pákozd* on the *Császárvíz* brook and in 1963 at *Kápolnásnyék* on the *Vereb—Pázmánd* brook, the second major watercourse in the catchment. The area drained by the Császárvíz brook down to the Pákozd gauge is 370 sq · km, whereas the catchment area of the Vereb—Pázmánd brook above the Kápolnásnyék gauge is 113 sq · km. [7].

The operational model

The model is intended to minimize the losses anticipated to occur as a consequence of undesirable low and high levels in Lake Velence. The basic data in the model include *the technical* data *of the reservoirs*, such as outlet capacity, highest and lowest levels, etc., further the time series of random changes in the reservoirs

$$\Delta W = P + R - E \tag{2}$$

where ΔW is the monthly change in water level, P is the monthly precipitation, R is the monthly runoff, and E is the monthly evaporation. Values of ΔW were obtained by direct observations on Lake Velence, or by simulation using a hydrological runoff model [7], also implying the quadratic *loss function* based on economic analyses for Lake Velence. This specifies the relative damages by months for periods with low or high stages, in terms of the deviation from the level considered as ideal. With these basic data the operational model is formulated using the graph in *Fig. 2* [9].

Adopting an arbitrary starting month (Month 1), the water volume stored in the i-th reservoir at the beginning of the k-th month is $T_i(k)$, while the release from the i-th reservoir during the k-th month is $r_i(k)$. Similarly $\Delta W_i(k)$ denotes the changes in water level during the k-th month (natural inflow, evaporation-seepage losses), considered as random variables.

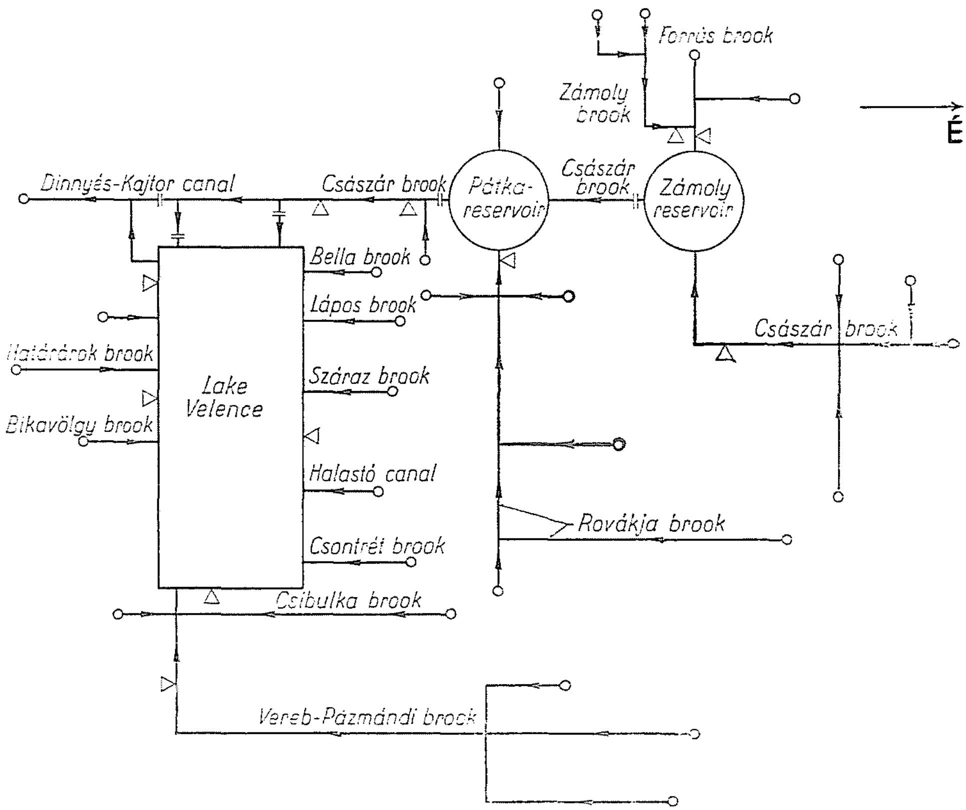

Fig. 2. Diagram of the water system of Lake Velence. Legend: △ water gauge;, ‖ sluice

According to the equation of continuity

$$T_i(k+1) = T_i(k) + \Delta W_i(k) + r_{i-1}(k) - r_i(k) \tag{3}$$

$(i = 1, 2, 3; k = 1, 2, \ldots, 12)$. Here the water volume released from the $(i-1)$-th reservoir during the month is assumed to reach the i-th reservoir by the beginning of the next month. From Eq. (3) we obtain:

$$T_i(2) = T_i(1) + \Delta W_i(1) + r_{i-1}(1) - r_i(1)$$

$$T_i(3) = T_i(1) + \sum_{s=1}^{2} \Delta W_i(s) + \sum_{s=1}^{2} r_{i-1} - \sum_{s=1}^{2} r_i(s)$$

and in general:

$$T_i(k) = T_i(1) + \sum_{s=1}^{k} \Delta W_i(s) + \sum_{s=1}^{k} r_{i-1}(s) - \sum_{s=1}^{k} r_i(s) \tag{4}$$

$(i = 1, 2, 3, k = 1, 2, \ldots, 12)$.

It is desired to maintain the water level in Lake Velence around the value considered as ideal. Specifically, the *criterion of the optimum* is formulated as follows: For given starting water supplies $T_i(1)$ $(i = 1, 2, 3)$, the releases $r_i(k)$ minimize the expected value of the actual monthly water volumes in Lake Velence relative to some constant monthly water volume V_k (the ideal water volume).
Mathematically

$$\bar{E}\left[\sum_{k=2}^{13} \varrho_k\{T_3(k) - V_k\}\right] \to \min. \tag{5}$$

Here the weighting numbers $\varrho_k (\varrho_{13} = \varrho_1)$ express the economic consideration that the same deviation from the optimum water volume is likely to cause different damages in different months.

$\varrho_i : \varrho_k = p$ i. e. damage incurred in the i-th month is p times that in the k-th month. Weighting numbers ϱ_k are thus dimensionless ratios and the problem in Eq. (5) is essentially to minimize the quadratic loss function.

Introducing the notation $\varphi_{i,k} = \sum_{s=1}^{k} \Delta W_i(s)$, Eq. (5) is rewritten:

$$\bar{E}\left[\sum_{k=1}^{12} \varrho_{k+1}\left\{T_3(1) + \varphi_{3,k} + \sum_{s=1}^{k} r_2(s) - \sum_{s=1}^{k} r_3(s) - V_k\right\}^2\right] \to \min. \tag{6}$$

For the decision variables, Eq. (6) represents a *quadratic programming problem* [10] where the quadratic part is

$$\sum_{k=1}^{12} \varrho_{k+1}\left\{\sum_{s=1}^{k} r_2(s) - \sum_{s=1}^{k} r_3(s)\right\}^2 \geqq 0$$

and its matrix is, therefore, semi-definite.

In solving Eq. (6) it is advisable and expedient to satisfy the following *constraints:*

I. $$0 \leq r_i(k) \leq L_i \qquad (i = 1, 2, 3;\ k = 1, 2, \ldots, 12) \tag{7}$$

where L_i denotes the maximum capacity of the outlet regulator of the i-th reservoir for a full month's duration.

II. $$S_{i\,\min} \leq T_i(k) \leq S_{i\,\max} \qquad (i = 1, 2, 3;\ k = 2, 3, \ldots, 13) \tag{8}$$

where $S_{i\,min}$ and $S_{i\,max}$ denote the minimum and maximum water volumes, respectively, to be stored in the i-th reservoir. For the Zámoly and Pátka reservoirs, these data are obtained from the physical characteristics, such as the minimum storage volume required for biological life, while in the case of Lake Velence, as the levels desired to be maintained, determined with a view on the actual interests.

III. The additional condition to be met:

$$r_i(k) \leq T_i(k) + r_{i-1}(k) + \Delta W_i(k) - S_{i\,\min}$$
$$(i = 1, 2, 3; \quad k = 1, 2, \ldots, \quad 12)$$

written into the form:

$$S_{i\,\min} \leq T_i(k) + r_{i-1}(k) + \Delta W_i(k) - \varphi_i(k) = T_i(k+1)$$

is thus satisfied, once (8) is. With Eq. (4), (8) becomes

$$S_{i\,\min} \leq T_i(1) + \sum_{s=1}^{k} r_{i-1}(s) - \sum_{s=1}^{k} r_i(s) - \varphi_{i,k} \leq S_{i\,\max} \tag{9}$$
$$(i = 1, 2, 3; \ k = 1, 2, \ \ldots, 12)$$

and since $\varphi_{i,k}$ is a random variable, in the case of Lake Velence and the related reservoir system the constraints (9) can only be met with certain probabilities $\alpha_{i,k}$, $\beta_{i,k}$ i. e., provided:

$$P\{T_i(k+1) \geq S_{i\,\min}\} \geq \alpha_{i,k}$$
$$(i = 1, 2, 3; \quad k = 1, 2, \ldots, \quad 12) \tag{10a}$$

$$P\{T_i(k+1) \leq S_{i\,\max}\} \geq \beta_{i,k} \,. \tag{10b}$$

For this reason the problem is that of a *chance constrained stochastic quadratic programming* [12, 13, 14, 15].

The process of optimization is as follows. The quantities φ_k, V_k are specified on economic considerations, while the probability levels $\alpha_{i,k}$ and $\beta_{i,k}$ so as to permit compliance with the sets (10a) and (10b). Considering further constraints (7) and the starting water volumes $T_i(1)$ $(i = 1, 2, 3; k = 1, 2, \ldots, 12)$ provides the optimum solution to the problem formulated in Eq. (6).

Of these the first release $(r_i(1))$, or the first few releases $(r_i(1), r_i(2), \ldots, r_i(k_0); (k_0 < 12))$ are only run and the procedure is repeated using ever new starting water volumes

$$\Delta W_i(1), \text{ or } \Delta W_i(1), \ \Delta W_i(2), \ \ldots, \ \Delta W_i(k_0).$$

Here again, only the first few releases are run and the procedure is continued.

The part of the model described in the foregoing applies to the case *where no forecast is available* and from the hydrologist's point of view it makes allowance essentially for the statistical behaviour of the data actually observed, or generated by analyzing random variables ΔW. In the computation form, expected value $\bar{E}(\Delta W_i)$ and the standard deviation $\sigma(\Delta W_i)$ of water level changes ΔW_i are used as hydrologic inputs to the model.

Conclusions

The operational model developed for the Lake Velence water resources system has been applied continuously in practice. As examples the result obtained for June, 1975 and 11 consecutive months are presented.

Operating instructions
(water volumes given in 10^5 m^3 units)

Releases computed by the model:

	Zámoly	Pátka	Lake Velence
June 1975	4.54	0	2.70
July	0	0	0
August	0	4.54	0
September	1.74	2.31	0
October	3.95	5.78	0
November	4.99	7 00	0
December	7.06	2.37	0
January 1976	0.78	0	6.06
February	0	0	30.35
March	0	0	11.51
April	0	0	15.99
May	0	0	22.63

The foregoing have led to the following fundamental conclusions:

1. The system model is of help in compiling instructions for the level regulation of Lake Velence best serving the recreation interests over extended periods of time.
2. The optimum operation of the water resources system consisting of the natural lake and the two upstream reservoirs can be determined by a chance-constrained stochastic quadratic programming method.
3. The solution of the model yields the monthly releases for one year. Of these the release for the month, or — depending on the actual hydrometeorological situation and water regime — even several ones can be realized, at a running time economy. In emergency situations the program can be re-run at 15-day intervals.
4. As to the allocation of water supplies, the model resembles in operation an "around-the-year reservoir", in that the surpluses from periods with ample runoff are stored for longer periods and utilized at times of low discharge.

Symbols

$\bar{E}$	Expected value
E[mm]	Multi-annual mean evaporation
H[mm]	Multi-annual mean inflow
L_i	Maximum monthly release capacity of the outlet gate of the i-th reservoir
P	Probability
$r_i(k)$, [10^5 m^3]	Water volume released from the i-th reservoir during the k-th month
$S_{i\,min}$, $S_{i\,max}$	Minimum and maximum storage capacities of the i-th reservoir
$T_i(k)$ [10^5 m^3]	Water volume stored in the i-th reservoir in the k-th month
$V_k \pm \Delta W$	Ideal water volume in Lake Velence in the k-th month
[mm or 10^5 m^3]	Natural change of water levels
$\alpha_{i,k}$, $\beta_{i,k}$	Probability levels
$\varphi_{i,k}$	Sum of natural changes in water level
ϱ_k	Weighting numbers, values of the economic loss function
σ	Standard deviation.

Summary

Water resources system of Lake Velence consists of a lake and two upstream reservoirs. The control model of the two reservoirs and the system has been developed for minimizing damage due to unduly low and high water levels in the lake. The problem was solved by computer running of a quadratic programming algorithm. The model involves a statistic preassessment to be completed in the future by meteorological forecasts. Continuous running of the model permits to issue monthly instructions for operating sluices on Lake Velence and its reservoirs.

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