

Shear Strength Prediction of FRP-reinforced Concrete Beams Using an Extreme Gradient Boosting Framework

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Abstract

Despite the importance and accuracy of empirical models, most of the existing models are only accurate on the collected experimental data. Adding new data, or even considering noise or variance in the data leads to loss of model accuracy. The objective of this paper is to alleviate overfitting and develop a more accurate and reliable alternative method using a decision-tree-based ensemble Machine Learning algorithm that uses a gradient boosting framework for the prediction of the ultimate shear strength of FRP-reinforced concrete beams without stirrups. To enhance the robustness of the results, make full use of training samples (without the validation set), and alleviate the randomness in selecting test samples, the K-Fold Cross Validation method is employed. Using a dataset including 205 samples, results show that the extreme gradient boosting framework (XGBoost) providing better prediction. In fact, XGBoost results have higher precision and higher generalization in comparison with the empirical equations, the current design codes of practice, Least Absolute Shrinkage and Selection Operator model (LASSO), and Random Forest model (RF).

Keywords

XGBoost, shear strength prediction, FRP, LASSO, Random Forest, Machine Learning

1 Introduction

The use of fiber-reinforced polymer (FRP) composites in strengthening reinforced concrete beams have been widely utilized for external flexural or shear reinforcement in construction. These materials are used as an alternative material for producing reinforcing bars. Some FRPs are non-conductive and non-corrosive helping durability in aggressive environments. Due to their mechanical properties including high longitudinal tensile strength and high fatigue endurance, and lightweight, in recent years, a large number of experimental tests have been conducted by researchers to determine the ultimate strength of reinforced concrete beams reinforced by FRPs.

The shear strength of FRP-reinforced concrete beams may differ from that of members reinforced with steel due to the difference in mechanical properties between FRP and steel reinforcement [1]. Shear failures were reported in several flexure tests for FRP-reinforced concrete beams including Michaluk et al. [2].

Razaqpur et al. [3] tested seven beams in bending to determine the concrete contribution to their shear resistance; they concluded that the ACI recommendations are

extremely conservative whereas the Canadian and Japan Society of Civil Engineers (JSCF) recommendations, albeit still conservative, are in closer agreement with the experimental data. The same conclusion is mentioned about ACI recommendations in [4]. Ashour [5] reported an inconsistency for the shear capacity calculation of GFRP-reinforced concrete beams between experimental results with those mentioned in ACI 318-99.

From a mechanics-based segmental approach, Zhang et al. [6] derived a generic closed-form solution for quantifying the shear capacity of RC beams and one-way slabs without stirrups.

Nowadays, Metaheuristics are used as a powerful tool for engineering prediction problems [7–10].

Some researchers used Artificial Neural Network (ANN) to solve prediction problems. For example, Kaveh et al. [11] used a deep learning approach for predicting the ultimate buckling load of variable-stiffness composite cylinders.

Because of the advancement of Machine Learning Regression methods (MLRs), in this paper, shear strength prediction of FRP-reinforced concrete beams has been

investigated using the most recent powerful MLRs including Least Absolute Shrinkage and Selection Operator (LASSO), Random Forest (RF), and extreme gradient boosting framework (XGBoost).

The remainder of this article is organized into five sections, as follows. Section 2 presents the basics of the Shear strength of FRP-RC beams and the dataset developed in this research. Section 3 is devoted to introducing the MLRs. This section includes a brief review of the K-Fold Cross Validation method. The results and discussion of the empirical equations, the current design codes of practice, and MLR results are presented in Sections 4. Finally, the conclusions are drawn in Section 5.

2 Shear strength of FRP-RC beams

Cracked reinforced concrete members resist the applied shear stresses by the following five mechanisms [12]:

- Shear stresses in uncracked concrete i.e., the flexural compression zone;
- Interface shear transfer (aggregate interlock/crack friction)
- Dowel action of the longitudinal reinforcing bars
- Arch action

- Residual tensile stresses transmitted directly across cracks

Researchers proposed several equations based on various experimental studies and the principles of structural mechanics. A summary of some shear design equations for FRP-reinforced members is provided in Table 1. As these equations have been empirically derived there are still gaps in selecting the main parameters affecting V_{cf} .

In this study, we utilized 24 datasets including 205 FRP reinforced concrete beams to benchmark our approach. These datasets are summarized in Table 2.

The dataset consists of 8 attributes including 7 input data features and 1 output. Table 3 and Fig. 1 present the summary of the statistical range of features and their corresponding correlations, respectively. In this figure relationships between random variables are provided. It can be inferred that the width of the web and the effective depth of the cross-section have the highest correlations with shear capacity.

Furthermore, the distribution of the parameters and corresponding shear strength included in the database is shown in Fig. 2.

Table 1 Shear design formulas for FRP-reinforced concrete beam without shear reinforcement

| Author(s)/Code | Formula | Note |
|-------------------------|--|--|
| ACI440.1R-15 [13] | $V_{cf} = \frac{2}{5} \sqrt{f'_c} b_w c$ | $c = kd$ $k = \sqrt{2\rho_f n + (\rho_f n)^2} - \rho_f n$ $n = \frac{E_f}{E_c}$ |
| CSA S806-02 [14] | For $d \leq 300$ mm $V_{cf1} = 0.35 \lambda \left(f'_c \rho_f E_f \frac{V_f}{M_f} d \right)^{1/3} b_w d$ For $d > 300$ mm: $V_{cf2} = \frac{130}{1000 + d} \lambda \sqrt{f'_c} b_w d$ | $V_{cf1(\min)} = 0.1 \lambda \sqrt{f'_c} b_w d$ $V_{cf1(\max)} = 0.2 \lambda \sqrt{f'_c} b_w d$ $V_{cf2(\max)} = 0.08 \lambda \sqrt{f'_c} b_w d$ |
| Tureyen and Frosch [15] | $V_{cf} = \frac{5}{12} k \sqrt{f'_c} b_w d$ | $k = \sqrt{2\rho_f n + (\rho_f n)^2} - \rho_f n$ $n = \frac{E_f}{E_c}$ |
| El-Sayed et al. [1] | $V_{cf} = \left(\frac{\rho_f E_f}{90 \beta_1 f'_c} \right)^{1/3} \left(\frac{\sqrt{f'_c}}{6} b_w d \right) \leq \frac{\sqrt{f'_c}}{6} b_w d$ | $\beta_1 = 0.85 - 0.05 \left(\frac{f'_c - 28}{7} \right) \geq 0.65$ |
| Alam and Hussein [16] | $V_{cf} = \frac{0.2 \lambda}{\left(\frac{a}{d} \right)^{2/3}} \left(\frac{\rho_f E_f}{d} \right)^{1/3} \sqrt{f'_c} b_w d$ | $V_{c(\min)} = \frac{0.1}{\left(\frac{a}{d} \right)} \lambda \sqrt{f'_c} b_w d$ $V_{c(\max)} = 0.2 \lambda \sqrt{f'_c} b_w d$ |

Table 2 Database of 205 FRP reinforced concrete beams

| Reference | Specimen |
|-----------------------------|--|
| Nagasaka et al. [17] | AN0090M, AN0090L |
| Nakamura and Higai [18] | G01, G02 |
| Zhao et al. [19] | 1, 2, 3 |
| Duranovic et al. [20] | GB2, GB6 |
| Swamy and Aburawi [21] | F3-GF |
| Deitz et al. [22] | GFRP1, GFRP2, GFRP3, Hybrid1, Hybrid2 |
| Yost et al. [23] | 1FRPa, 1FRPb, 1FRPc, 2FRPa, 2FRPb, 2FRPc, 3FRPa, 3FRPb, 3FRPc, 4FRPa, 4FRPb, 4FRPc, 5FRPa, 5FRPb, 5FRPc, 6FRPa, 6FRPb, 6FRPc |
| Alkhrdaji et al. [24] | BM7, BM8, BM9 |
| Massam [25] | LB/8/0.5/0, LB/4/0.5/0, LB/2/0.5/0, LB/8/2/0, LB/4/2/0, LB/2/2/0 |
| Tureyen and Frosch [4] | V-G1-1, V-G2-1, V-A-1, V-G1-2, V-G2-2, V-A-2 |
| Tariq and Newhook [26] | G07N1, G07N2, G10N1, G10N2, G15N1, G15N2, C07N1, C07N2, C10N1, C10N2, C15N1, C15N2 |
| Gross et al. [27] | FRP-1a-26-NS, FRP-1b-26-NS, FRP-1c-26-NS, FRP-2a-26-NS, FRP-2b-26-NS, FRP-2c-26-NS, FRP-3a-36-NS, FRP-3b-36-NS, FRP-3c-36-NS, FRP-4a-46-NS, FRP-4b-46-NS, FRP-4c-46-NS, FRP-5a-37-NS, FRP-5b-37-NS, FRP-5c-37-NS, FRP-6a-37-NS, FRP-6b-37-NS, FRP-6c-37-NS |
| Razaqpur et al. [3] | BR1, BR2/BA2, BR3, BR4, BA1, BA3, BA4 |
| El-Sayed et al. [28] | S-C1, S-C2B, S-C3B, S-G1, S-G2, S-G2B, S-G3, S-G3B |
| Ashour [5] | Beam1, Beam3, Beam5 |
| El-Sayed et al. [29] | CN-1, GN-1, CN-2, GN-2, CN-3, GN-3 |
| Guadagnini et al. [30] | GB43, GB44, GB45 |
| Matta et al. [31] | SI-1, S3-2, S3-3, S6-2, S6-3, S1B-1, S1B-2 |
| Jang et al. [32] | C-1.5-R1-1-2, C-1.5-R2-1-2, C-1.5-R3-1-2, C-2.0-R1-1-2, C-2.0-R2-1-2, C-2.0-R3-1-2, C-2.5-R1-1-2, C-2.5-R2-1-2, C-2.5-R3-1-2, C-3.5-R1-1-2, C-3.5-R2-1-2, C-3.5-R3-1-2, C-4.5-R1-1-2, C-4.5-R2-1-2, C-4.5-R3-1-2, G-1.5-R1-1-2, G-1.5-R2-1-2, G-1.5-R3-1-2, G-2.0-R1-1-2, G-2.0-R2-1-2, G-2.0-R3-1-2, G-2.5-R1-1-2, G-2.5-R2-1-2, G-2.5-R3-1-2, G-3.5-R1-1-2, G-3.5-R2-1-2, G-3.5-R3-1-2, G-4.5-R1-1-2, G-4.5-R2-1-2, G-4.5-R3-1-2 |
| Bentz et al. [33] | M05-0, S05-0, M20-0, S20-0, I05-0, I20-0 |
| Olivito and Zuccarello [34] | SI-1, SI-2, SI-3, SI-4, SI-5, SII-1, SII-2, SII-3, SII-4, SII-5, SIII-1, SIII-2, SIII-3, SIII-4, SIII-5, SIV-1, SIV-2, SIV-3, SIV-4, SIV-5 |
| Abed et al. [35] | B1FRP, B2FRP, B3FRP, B4FRP, B5FRP, B6FRP, B7FRP |
| Alam and Hussein [16] | G-2.5, G-3.5, C-2.5, C-3.5, G-500, G-650, G-800, C-500, C-650, C-800, G-2.5-350, G-2.5-350, G-0.5-500, G-2.5-500, C-0.5-350, C-2.5-350, C-0.5-500, C-2.5-500 |
| Matta et al. [36] | SI-0.12-1A, SI-0.12-2B, S3-0.12-1A, S3-0.12-2A, S6-0.12-2A, S6-0.12-3A, SI-0.24-1A, SI-0.24-2B, S3-0.24-1B, S3-0.24-2B, S6-0.24-1B, S6-0.24-2B |

Table 3 Statistical range of material and geometric properties

| Property | Minimum | Maximum | Mean | Standard deviation |
|------------------|---------|---------|---------|--------------------|
| b (mm) | 114.00 | 1000.00 | 260.92 | 177.54 |
| d (mm) | 146.00 | 938.00 | 286.27 | 169.28 |
| a/d | 1.00 | 6.50 | 3.59 | 1.23 |
| f'_c (MPa) | 22.70 | 50.00 | 34.98 | 5.63 |
| ρ_{FRP} (%) | 0.18 | 3.43 | 1.12 | 0.63 |
| F_{sy} (MPa) | 397.00 | 2250.00 | 1209.56 | 615.88 |
| E_f (GPa) | 22.70 | 30.40 | 26.47 | 1.59 |
| V_{exp} (KN) | 12.50 | 232.00 | 64.81 | 51.60 |

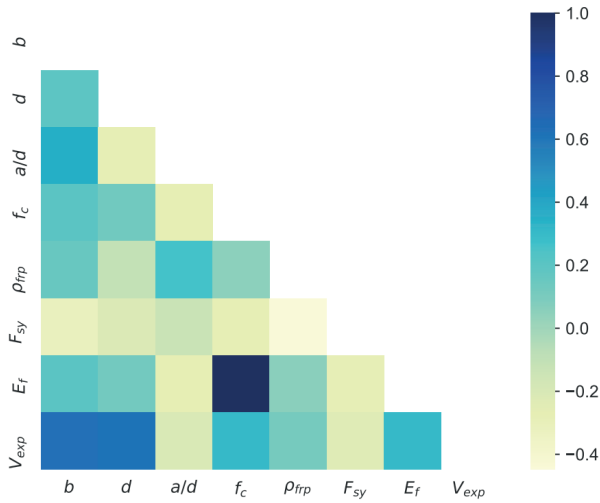


Fig. 1 Correlation of the features

3 Machine Learning prediction

3.1 Least Absolute Shrinkage and Selection Operator (LASSO)

LASSO [37] is an MLR analysis method. In this method, in order to avert overfitting, the linear model is trained with L_1 prior as a regularizer. In other words, LASSO adds an absolute value of the magnitude of coefficient as a penalty term to the loss function. In this model, the optimization function defined as:

$$\text{Loss function} = \sum_{i=1}^n (y_i - (\beta_0 + \sum_{j=1}^p \beta_j x_{ij}))^2 + \lambda \sum_{j=1}^p |\beta_j|, \quad (1)$$

where x_{ij} is the training data and y_i is the observation. In the above loss function, the first part is the residual sum of squares and the remaining is the sum of the absolute value of coefficients. In addition, λ is the penalty parameter providing a trade-off between the first and second parts. lasso imposes an L_1 penalty on the β . That is, lasso finds an assignment to β_0 and β that minimize the loss function.

3.2 Random Forest (RF)

Random Forest regression as a supervised machine learning algorithm uses an ensemble learning method for prediction. The RF model for regression can be described in Algorithm 1.

An illustration of ensemble learning for RF is shown in Fig. 3. The RF algorithm is not biased and it improves stability and accuracy because of the combination of several trees. Further details of the RF algorithm can be found in [38].

3.3 Extreme gradient boosting framework (XGBoost)

XGBoost is an efficient implementation of gradient boosting that can be used for regression predictive modeling.

XGBoost as a gradient boosting decision tree adds models sequentially; however, random forest adds multiple predictors in parallel. It utilizes regularized learning and cache-aware block structure tree learning. This method is based on function approximation and optimizing specific loss functions expressed as follows [39]:

$$\mathcal{L}^t = \sum_{i=1}^n l(y_i, \hat{y}_i^{(t-1)} + f_t(x_i)) + \Omega(f_t), \quad (2)$$

$$\Omega(f) = \gamma T + \frac{1}{2} \lambda \omega^2, \quad (3)$$

where l is a differentiable convex loss function measuring the difference between the \hat{y}_i and the y_i . The second term Ω penalizes the complexity of the model in order to prevent overfitting. T is the number of leaves and ω is

Algorithm 1 Random Forest for regression

For $i = 1$ to T :

- (a) Take a bootstrap sample of size k from the training data.
 - (b) Grow an RF tree to the bootstrapped data:
 - i. Select m variables at random out of all M
 - ii. Find the best split on the selected m variables
 - iii. Grow the trees
 - (c) Average the results from all the T trees
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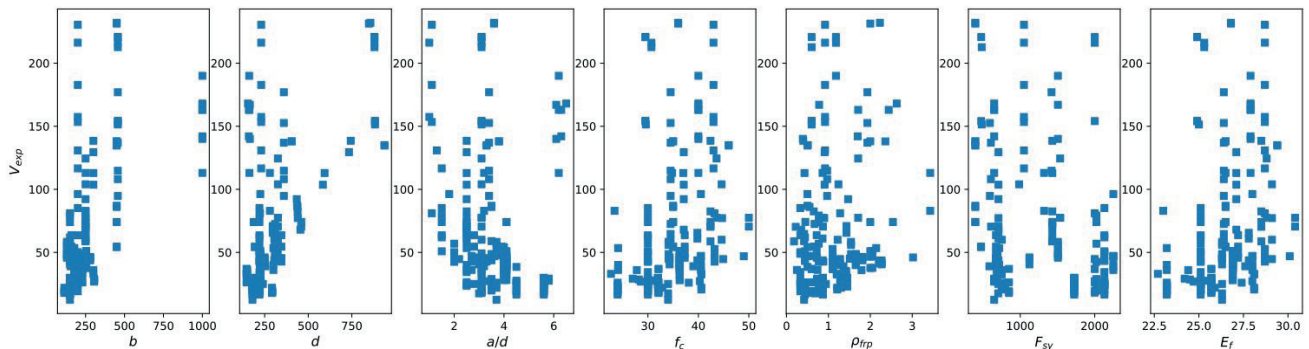


Fig. 2 The relationship between the seven input variables and the nominal shear strength provided by concrete in the data set

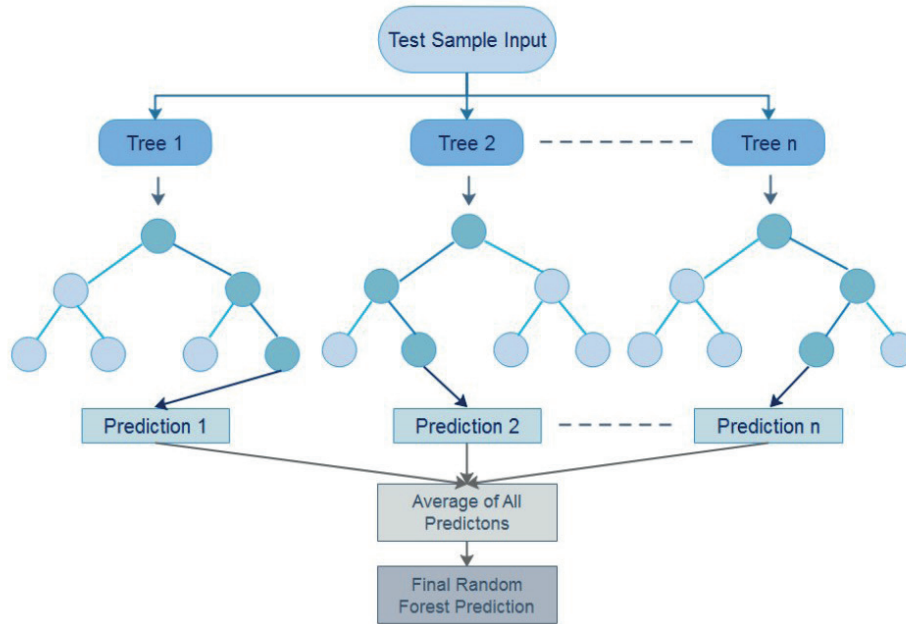


Fig. 3 Illustration of ensemble learning for random forest

the vector of scores on leaves. By using the second-order Taylor approximation, Eq. (2) can be expressed as follows:

$$\mathcal{L}^{\prime}(q) = -\frac{1}{2} \sum_{j=1}^T \frac{(\sum_{i \in I_j} g_i)^2}{h_j + \lambda} + \gamma T, \quad (4)$$

$$g_i = \partial_{\hat{y}^{(t-1)}} l(y_i, \hat{y}_i^{(t-1)}), \quad (5)$$

$$h_i = \partial_{\hat{y}^{(t-1)}}^2 l(y_i, \hat{y}_i^{(t-1)}). \quad (6)$$

To avoid overfitting, XGBoost integrates multiple weak predictors rather than strong ones. XGBoost uses scalability in all scenarios and handles sparse data using a tree learning algorithm. Further details about this algorithm can be found in [39].

3.4 K-Fold Cross-Validation

In a supervised machine learning problem, holding out part of the data as a validation set is prevalent. However, to avoid overfitting and the need to make full use of samples that can be used for training the model, the K-Fold Cross Validation [40] is used in this paper. By applying this method, the validation set is no longer needed. Here, the data set is split into 10 folds. In the first iteration, the first fold is used to test the model and the remainder is used to train the model. This process is repeated until each fold of the 10 folds has been used as the testing set. Fig. 4 illustrates the general structure of the 10-Fold Cross-Validation used in this investigation. The flowchart for the ML model is illustrated in Fig. 5.

4 Results of Prediction models

For our investigation, the entire dataset is divided into two parts, 80% for training and 20% for testing. In this paper, the width of the web, effective depth of cross-section, shear span to effective depth ratio, the specified compressive strength of concrete, FRP reinforcement ratio, modulus of elasticity of flexural FRP reinforcement, and yielding strength of the reinforcement have been taken as the input variables.

4.1 Performance measures

The results of the ML models are compared using the root-mean-square error (RMSE), mean absolute error (MAE), and R-squared (R²). Definitions of these statistical indicators are given as follows:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2}, \quad (7)$$

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|, \quad (8)$$

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}, \quad (9)$$

where N is the number of testing samples, \hat{y}_i and the y_i are the target and the prediction of the i -th sample, respectively; \bar{y} is the average of the predicted outputs.

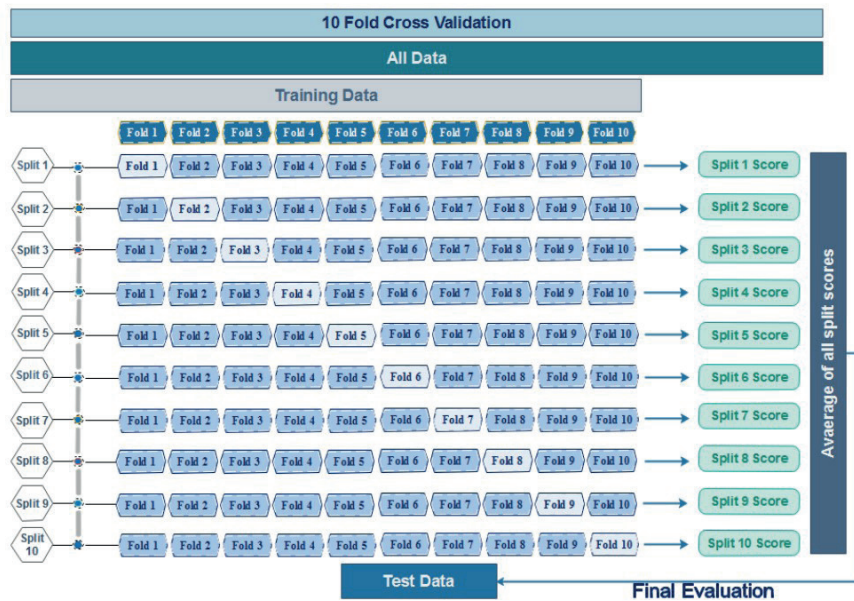


Fig. 4 Illustration of 10-Fold Cross-Validation

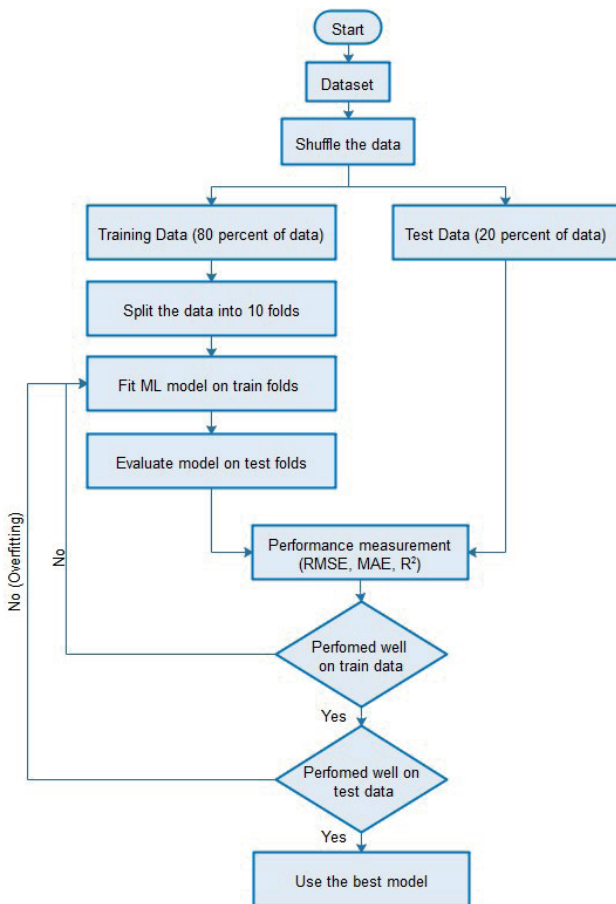


Fig. 5 Implementation of ML model

4.2 Comparison with previous Modeling Approaches

A feature importance analysis is performed for the XGBoost model to break down the seven input variables for the shear strength of FRP-reinforced concrete beams, and the results

are shown in Fig. 6. As it can be seen, the four variables determined to have high feature importance, i.e., the width of the web, effective depth of cross-section, shear span to effective depth ratio, and specified compressive strength of concrete. In this analysis, the modulus of elasticity of flexural FRP reinforcement has the smallest relative importance.

Fig. 7 Shows the comparison of prediction models for shear capacity with experimental data. By comparing the experimental and the theoretical model for 40 test samples, it is inferred that current guidelines provide conservative values. On the other hand, the XGBoost and RF model outperforms LASSO and the other empirical models.

The values of the RMSE, R^2 , and MAE are presented in Fig. 8. Results indicate that the XGBoost predicts the experimental shear capacity relatively accurately. In addition, it can be inferred that XGBoost outperforms other MLR methods; on the basis of the RMSE and MAE, the

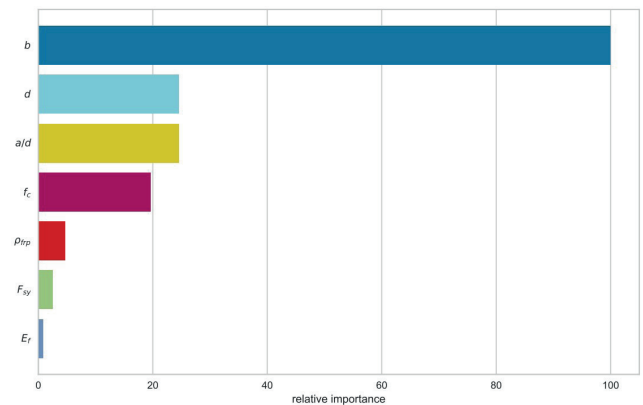
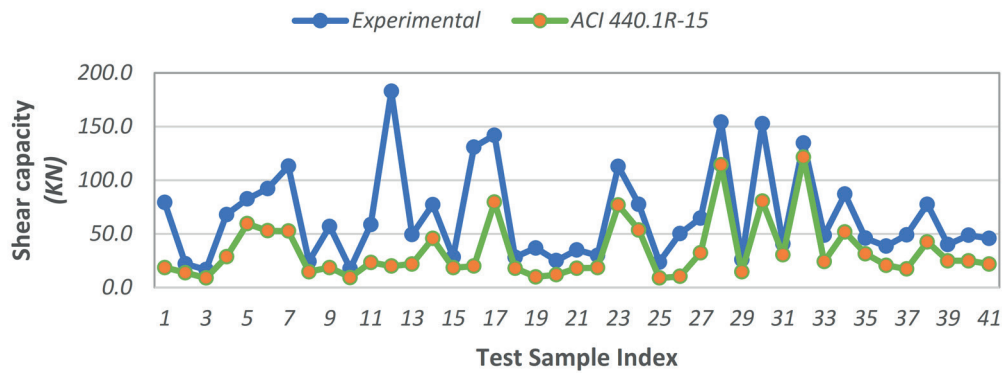
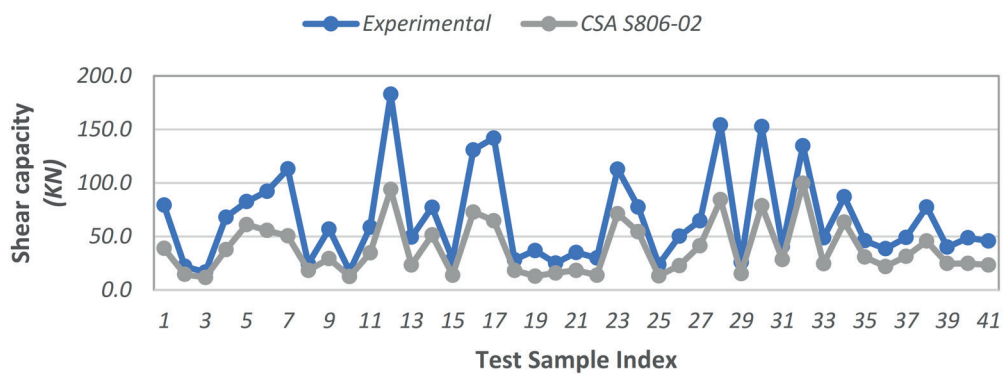


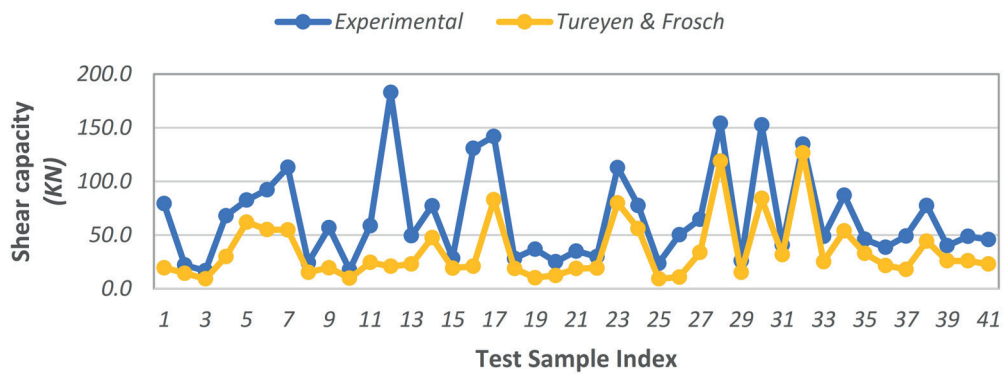
Fig. 6 Feature importance of the seven input variables using XGBoost



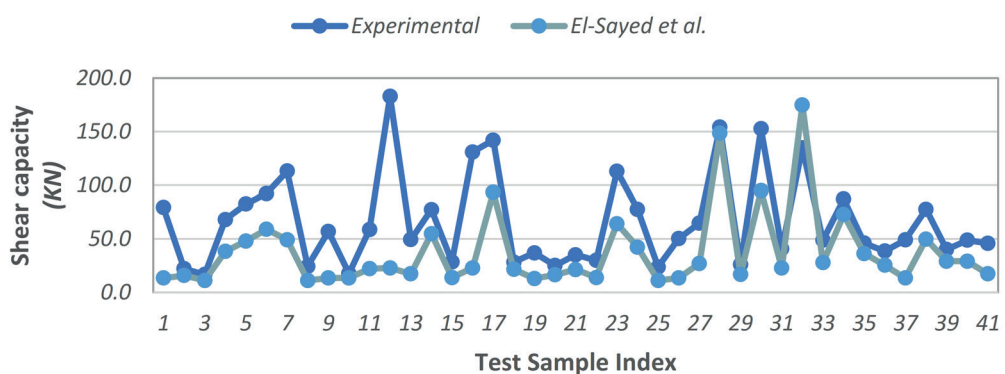
(a)



(b)

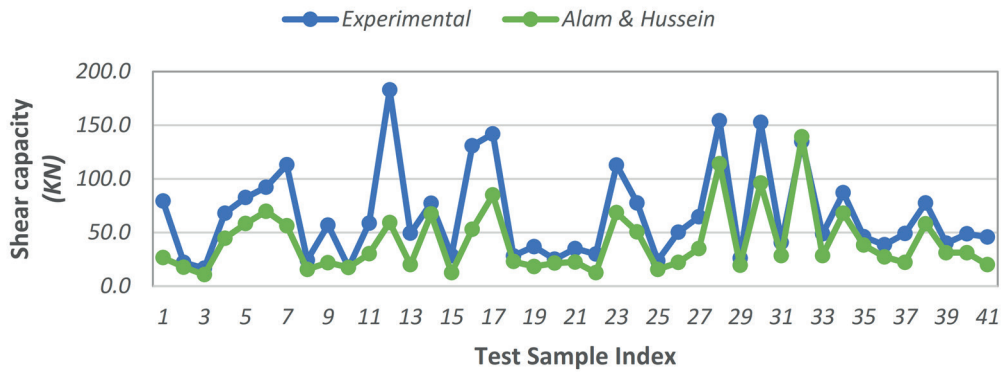


(c)

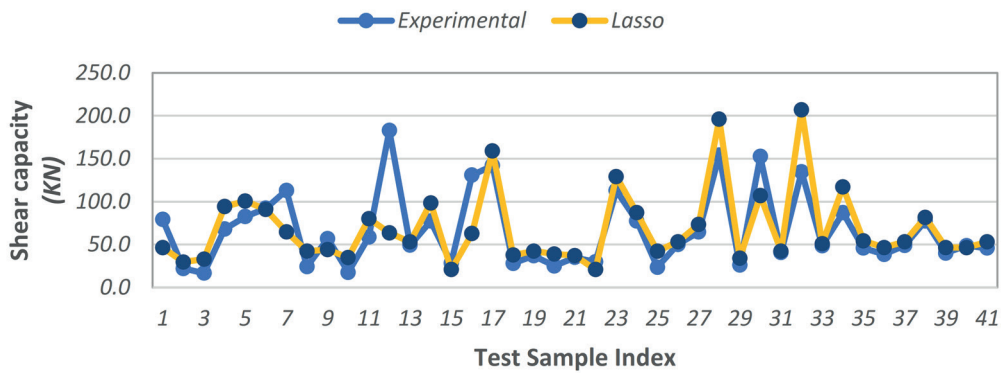


(d)

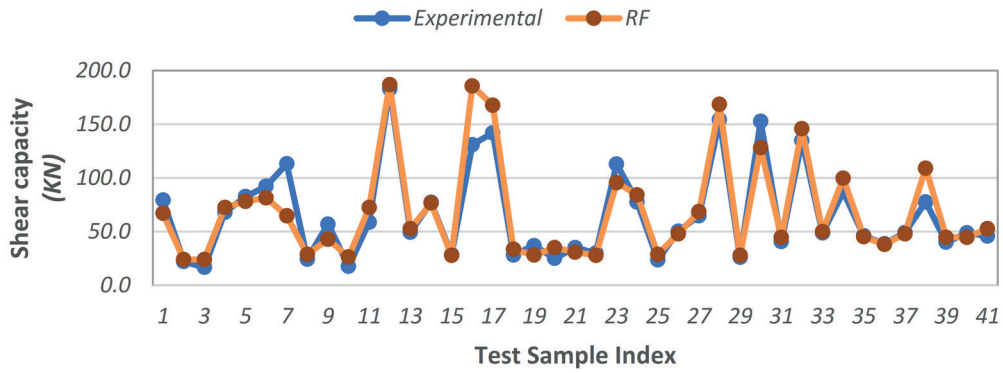
Fig. 7 Comparison of predicted shear capacity with experimental shear resistance (Figs. (a) to (d))



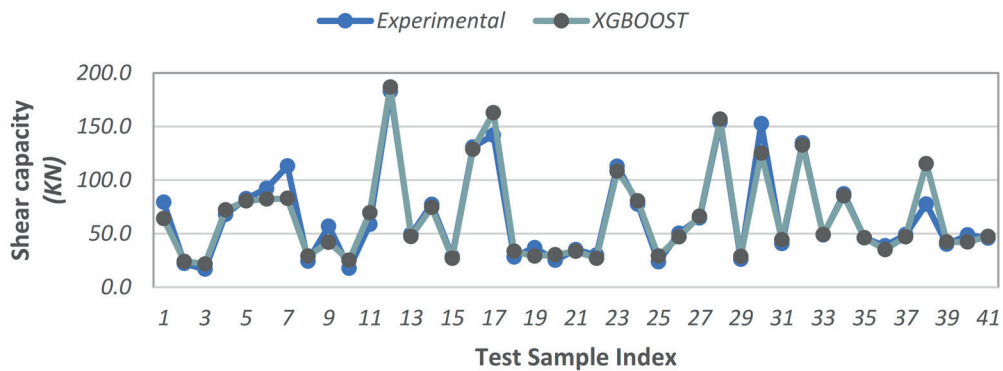
(e)



(f)



(g)



(h)

Fig. 7 Comparison of predicted shear capacity with experimental shear resistance (Figs. (e) to (h))



Fig. 8 Results comparison between all test samples for shear strength prediction of FRP-reinforced concrete beams: (a) RMSE; (b) MAE; (c) R²

XGBoost model obtains RMSE of 10.66 KN, and MAE of 6.67 KN which are lower than RF, LASSO, empirical equations, and the design codes of practice used in this paper.

The differences between the experimental values and the predicted values for the XGBoost model are shown in Fig. 9. The residuals plot shows the difference between residuals on the vertical axis and the predicted value on the horizontal axis. Due to the lack of sufficient experimental data, it can be inferred that regions with a shear strength of larger than are susceptible to more error.

5 Conclusions

The primary aim of this study is to use the ensemble learning framework to improve the shear strength prediction of FRP-reinforced concrete beams without stirrups. A relatively large dataset with over 205 samples is collected from different research papers and used to train the predictive models.

The paper details the feasibility of using MLR methods including LASSO, RF, and XGBoost algorithm for prediction problems. The 10-Fold Cross-Validation technique is used to improve the robustness of the results, make full

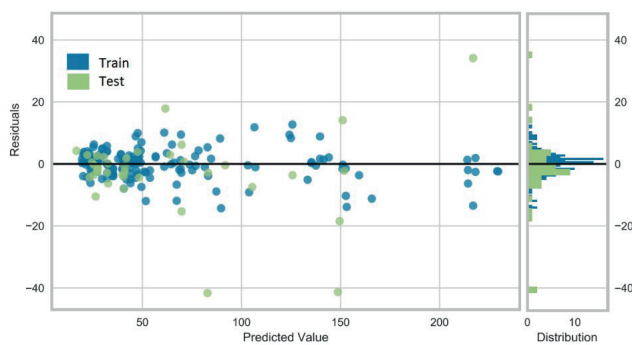


Fig. 9 Residual values in regression analysis for the XGBoost model

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use of training samples, and alleviate the randomness in selecting test samples. Results of different performance metrics are taken into account. Results show that the extreme gradient boosting framework has successfully predicted the shear strength with a high degree of accuracy and outperforms the other previously developed models. On the other hand, it is inferred that current guidelines provide conservative values of the concrete shear strength of FRP-reinforced concrete beams.

Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

Nomenclature

- a shear span
- b_w width of the web
- d the effective depth of the cross-section
- E_c modulus of elasticity of concrete
- f'_c specified compressive strength of concrete
- c distance from extreme compression fiber to the neutral axis
- f_{sy} yielding strength of the reinforcement
- E_f modulus of elasticity of flexural FRP reinforcement
- V_{cf} nominal shear strength provided by concrete
- ρ_f reinforcement ratio
- λ modification factor for the density of concrete
- β_1 the depth reduction factor of the equivalent rectangular stress block

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