RESONANCE EFFECTS IN THE HYDRAULIC TRANSIENT BEHAVIOUR OF SHOWER TRAY COLUMNS

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Distillation columns are often controlled by manipulating the boilup rate through the heat input to the reboiler. In some cases this may be more advantageous than manipulating the reflux rate because vapour rate changes propagate with greater speed, than do liquid rate changes. Estimation of the value of hydraulic lag would be essential for the design of distillation control, but knowledge especially on vapour flow lag is deficient.

Time lag in vapour flow rate changes in distillation columns may be attributed to the following effects.

1. Vapour spaces between the trays respond to vapour rate changes as a series of pressure vessels (interacting first-order elements). The resulting time lag is generally negligible.

2. In transient operation, part of the increased vapour flow must condense to heat the tray holdup and the tray material to the higher saturation temperature due to increased pressure drop. The resulting delay on the 20th tray (from bottom) of a benzene–toluene fractionator was estimated by HARRIOTT as 1 to 2 minutes [1].

3. An interaction between column pressure drop changes accompanying vapour rate variations and boiling temperature of the reboiler liquid holdup has been found by HAJDÚ, BORUS and FÖLDES [2]. This effect was shown to produce quite important vapour flow lags of several minutes in the case of kettle type reboilers. Effect 3 has been investigated for the case of downcomer trays [2]. Here liquid holdup may be assumed as independent of vapour rate, accordingly both the pressure drop response of a tray to a vapour rate change and (provided effects 1 and 2 are negligible) the pressure drop response of the whole column are instantaneous and behave as a proportional element. The whole process is shown in the block diagram of Fig. 1 indicating also the simplified transfer function of the reboiler. The complete process i. e. the response of vapour production in the reboiler to a change in the heat input, could be described by that of a first-order element.

In the present work, the same effect is investigated for the case of shower tray columns (trays without downcomers). This case needs a more

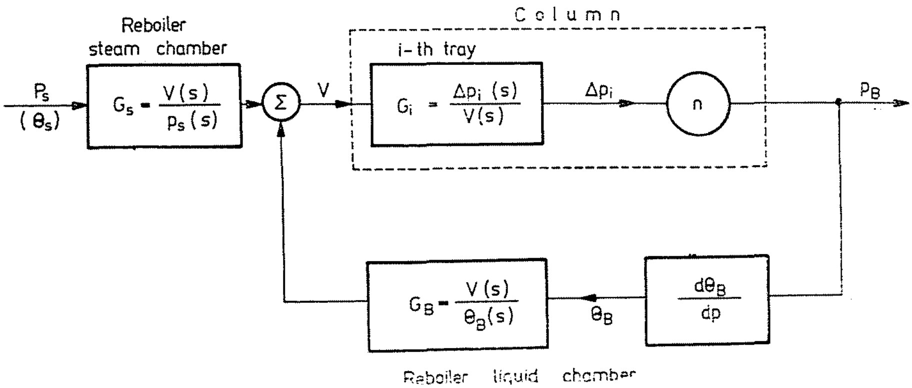

Fig. 1. Scheme of the interaction between column pressure drop and rebo ler heat transfer for downcomer trays

sophisticated model, because tray holdups on shower trays depend strongly on flow rates of both the vapour and liquid. As a first step, a model for the pressure response in the bottom of an n-tray column is developed based on the tray response model of MOLNÁR and FÖLDES [3]. Frequency functions and transient response for step disturbance are computed.

Tray response to liquid rate variations [3].

The liquid holdup varies, if the liquid rates entering and leaving the tray differ. Material balance on the i-th tray gives:

$$\frac{dH_i}{dt} = L_{i-1} - L_i \tag{1}$$

MOLNÁR and FÖLDES [3] in their hydraulic model for turbogrid trays replace H_i and L_i by the directly measurable tray pressure drop and assume a linear dependence within the range of variation:

$$\Delta p_i = A + aH_i \ , \tag{2}$$

$$\Delta p_i = B + bL_i \ . \tag{3}$$

With Eqs (2) and (3), Eq. (1) is written as:

$$\frac{d(\Delta p_i)}{dt} = \frac{a}{b}(\Delta p_{i-1} - \Delta p_i) \tag{4}$$

showing that the pressure drop on a turbogrid tray responds to a liquid rate disturbance as a first-order element. Accordingly, the i-th tray from the liquid rate upset responds as an i-th order element; its transfer function being (if all the trays are similar):

$$G_i(s) = \frac{K}{(Ts+1)^i} \tag{5}$$

Where

$$K = \frac{\partial(\Delta p_i)}{\partial L} = b \quad \text{is the gain factor,}$$

$$T = \frac{b}{a} \quad \text{is the time constant.}$$

Tray response to vapour rate variations [3].

MOLNÁR and FÖLDES found [3] the tray pressure drop response on turbogrid trays to vapour rate disturbance to consist of two parts. The first part of the response is practically simultaneous with the disturbance and is effected by the passage of the increased amount of vapour through the tray. This first part of the pressure drop change is negligible compared with the second part caused by the change of the liquid holdup due to vapour rate change. The mechanism of this part of the response is that a sudden increase in the vapour rate simultaneously reduces the liquid downflow from the tray and provided the liquid rate entering the tray is constant the liquid holdup (hence the static liquid head and the pressure drop) will build up to a value ensuring equal onflow and downflow of the liquid. If vapour lag effects 1 and 2 are negligible, then in an n-tray column the vapour rate change and concomitant liquid rate change occur simultaneously on every tray, only the liquid onflow to the top tray remains unaltered.

Thus, the pressure drop response on any tray is essentially the same both for vapour rate and liquid rate disturbances, Eq. (4) being valid. The transfer function is Eq. (5) for both cases, the first tray being the top tray.

Pressure response in the reboiler

The pressure response in the reboiler is the sum of pressure drop responses on all the trays (provided the top pressure is constant), thus the transfer function of the reboiler pressure for both vapour and liquid rate variations is sum of the tray transfer functions:

$$G(s) = \sum \frac{\Delta p_i(s)}{V(s)} = \frac{K}{Ts+1} + \frac{K}{(Ts+1)^2} + \ldots + \frac{K}{(Ts+1)^n} \tag{6}$$

provided the gain factors and the time constants are equal for all trays.

The block diagram of the process for vapour rate variations is shown in Fig. 2.

The frequency function may be computed from the normalized transfer function:

$$G(j\omega) = \frac{G(j\omega)}{K} = \frac{1}{Tj\omega + 1} + \frac{1}{(Tj\omega + 1)^2} + \ldots + \frac{1}{(Tj\omega + 1)^n} \quad (7)$$

using a recursion formula for n trays.

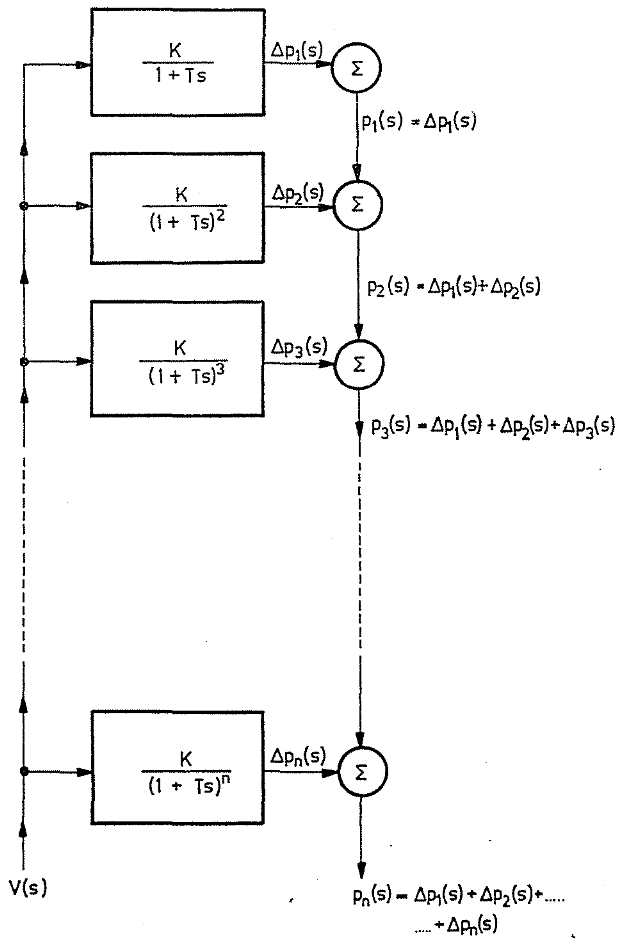

Fig. 2. Block diagram of the process for vapour rate variations in the case of an n-tray column

Computed results

The frequency functions of column pressures up to 100 trays were computed. The results have been plotted as both Bode and Nyquist diagrams.

The Bode diagram of a column with five trays is practically identical with that of a first-order element with gain and time constant five times those of one tray.

The Bode diagram for columns up to 15 trays differ from the behaviour of first-order elements especially in the region of moderate frequencies, but it can still be satisfactorily approximated by a first-order element with a gain factor and a time factor n times those of a single tray (see in Fig. 3 for $n = 10$).

For columns with 20 or more trays, resonance effects appear in both the Bode diagrams (Fig. 4) and the Nyquist diagrams (Fig. 5). For zero frequency, the gain factor is n times that of a single tray. Increasing the frequency of the vapour (or liquid) rate disturbance, a periodic decrease and increase of the gain factor and of the phase lag occur as a function of the total tray number and

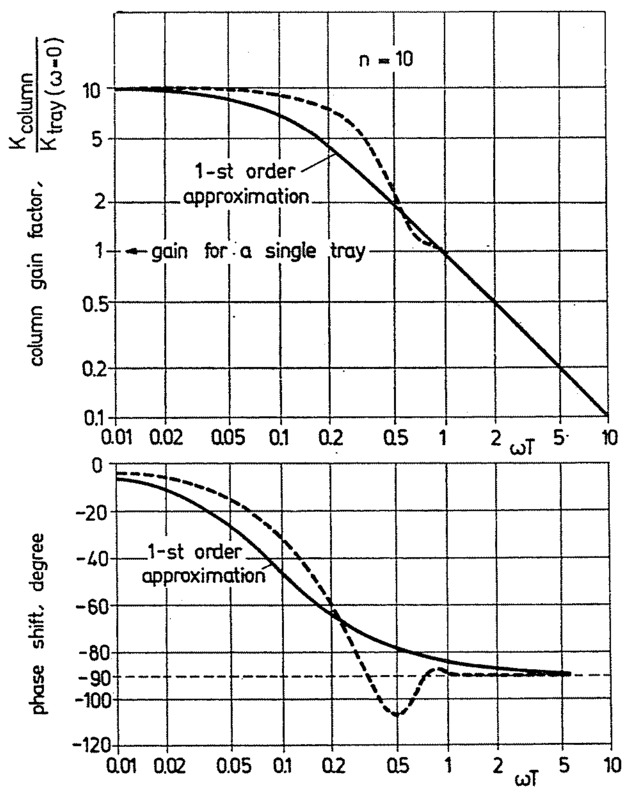

Fig. 3. Computed Bode diagram for a ten-tray column and its approximation with $\tau = 10T$

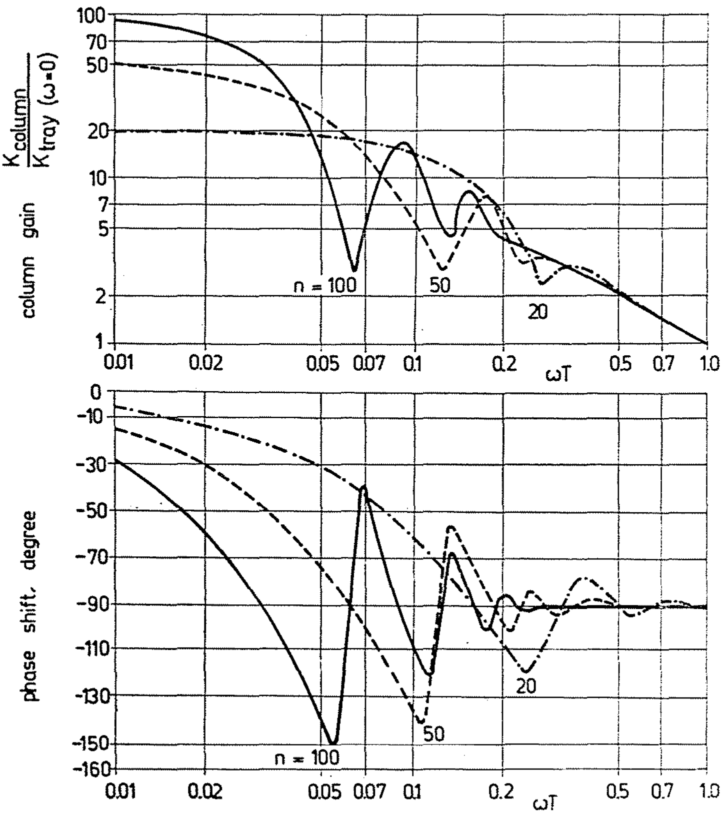

Fig. 4. Computed Bode diagram for 20-; 50-; 100-tray columns

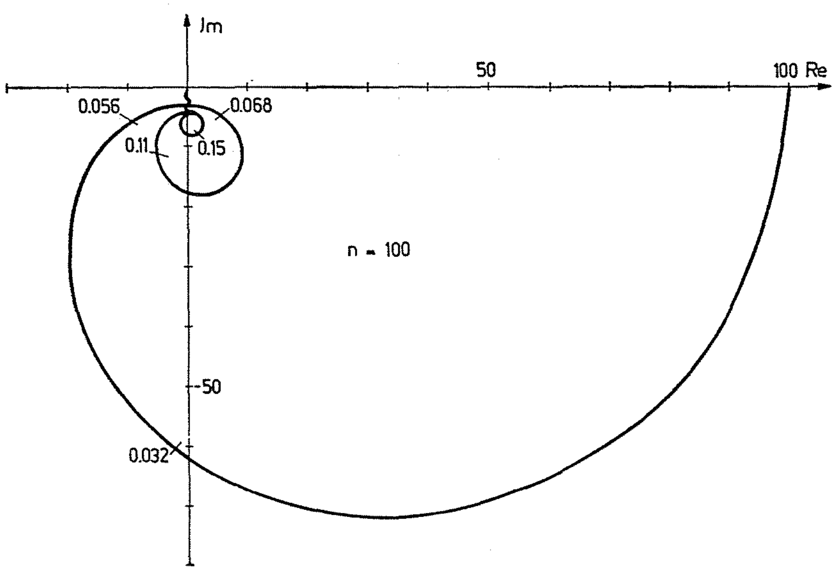

Fig. 5. Computed Nyquist diagram for a 100-tray column

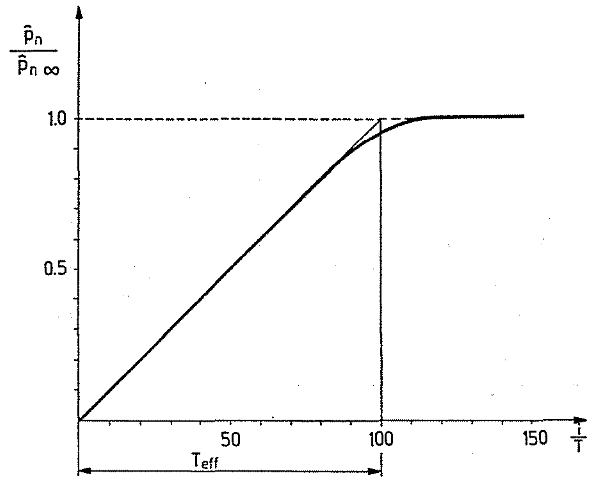

Fig. 6. Step response of a 100-tray column

the frequency. The minimum phase lag is a function of the tray number: for a 50-tray column it is about 140 degrees, for a 100-tray column about 150 degrees. When the frequency tends to infinity, both the gain and the phase lag approach those of a single tray, independent of the actual tray number.

The transient response for the case of step disturbance was also computed — as sum of the transient responses of the trays. The response is similar to that of a first-order element for small tray numbers of about five in accordance with the experimental results of MOLNÁR and FÖLDES with air–water system [3], and of BORUS (unpublished) with water–water vapour system, in a pilot column with five turbogrid trays. The computed step response of a 100-tray column is represented in Fig. 6. The effective time constant determined from the response was $T_{eff} = 100T$.

Discussion

The resonance effects involved in the frequency functions may be attributed to the fact that the pressure under the i-th tray is the sum of pressure drops produced on all the i trays and these pressure drops are in different phases of response to the sinusoidal forcing. Fig. 7 shows the propagation of the pressure drop wave down a 20-tray column upon sinusoidal forcing with the frequencies resulting in column gain maximum and minimum (see also Fig. 4). Depending on the frequency of forcing, the majority of the tray pressure drops may be greater or smaller than the initial one before forcing. Thus the process gain is increased or decreased producing the peaks and valleys in the frequency response.

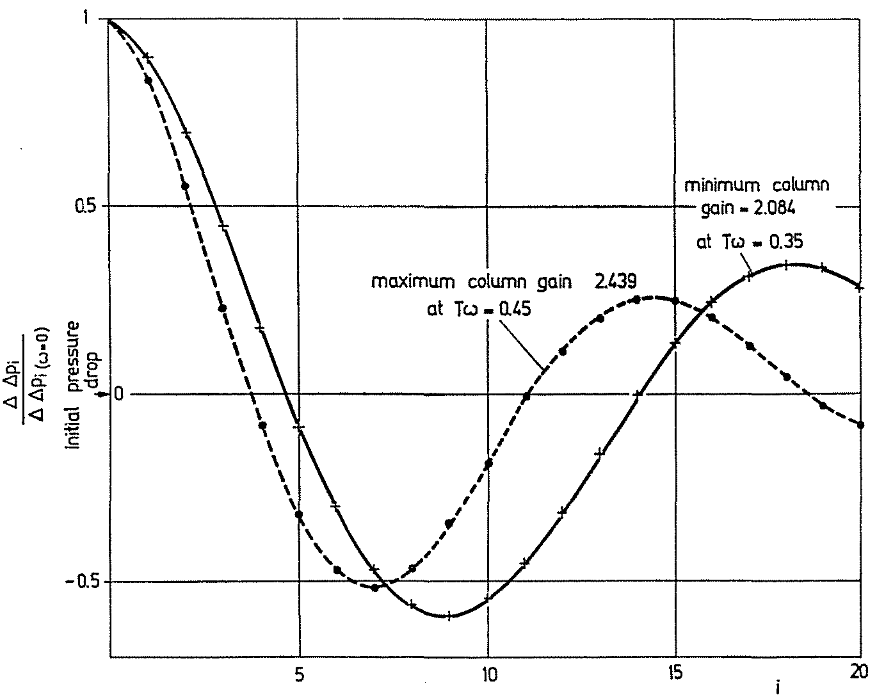

Fig. 7. The pressure drop wave on individual trays downwards in a 20-tray column as a response to sinusoidal forcing at frequencies resulting in maximum, and minimum column gain

The resonance effects can also be interpreted by the existence of side capacities. Since for both vapour and liquid rate disturbances, pressure drop variations are induced by liquid holdup changes, the top tray acts as a first-order element, all the other trays respond as parallel connected higher order elements and are not reached by the forcing in the limiting case of infinite frequency.

The computation of the process transfer function and rise of the resonance are illustrated in the complex plane in Fig. 8. Here $\sum i$ denotes the resulting complex vector of the pressure transfer function under the i-th tray, $(i + 1)$ denotes the complex vector of the pressure drop transfer function through the $(i + 1)$-th tray and $\sum(i + 1)$ the sum vector of the pressure transfer function under the $(i + 1)$-th tray. With increasing frequency the phase lag of the higher order trays may surpass 360° or its multiples but in the vectorial sum only the principal values appear. This is the reason of the periodic fluctuation of the phase lag values.

Also the frequency functions of shell and tube heat exchangers are known to show resonance effects [1]. The analogy is, however, not complete, mainly because in heat exchangers the temperatures of the fluid particles are in an

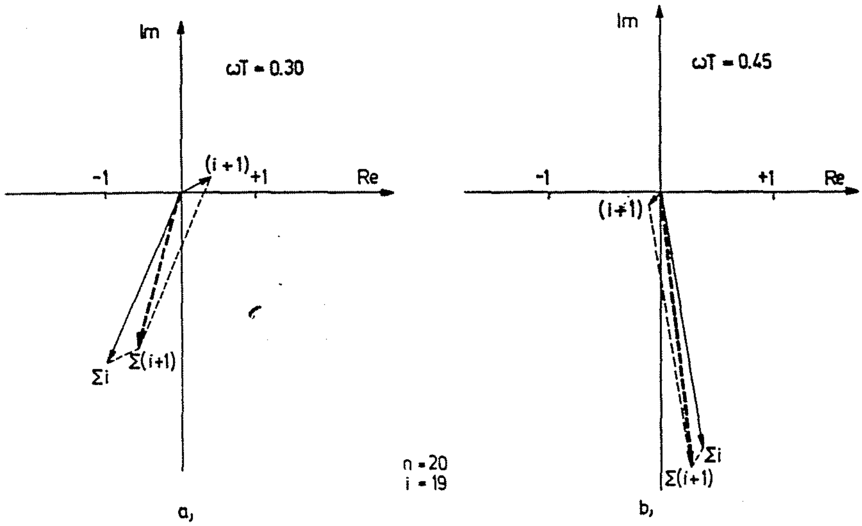

Fig. 8. Illustration of the resonance in the process frequency function on the complex plane
a) Decreasing column gain and phase lag
b) Increasing column gain and phase lag

interaction in both directions of the fluid flow, whereas in tray columns the action points downwards from the top tray: the bottom tray has no influence on the trays above it.

Our theoretical results are supported by experiments carried out on a 1 m diameter ethane-ethylene fractionating unit with 91 turbogrid trays reported by RADEMAKER [4]. He applied two kinds of forcing: step disturbance and sinusoidal disturbance; the manipulated variables were: the pressure of the heating medium to the reboiler and the top pressure. Pressure drop through the column was measured. Using the reported Bode diagrams a tray time constant of 2 s can be computed from the frequencies producing phase lag minima and maxima which is a realistic value for similar trays. RADEMAKER attributes the "rather peculiar" form of the Bode diagram to the interaction between liquid and vapour flow, but gives no theoretical explanation.

Conclusion

Previously, the pressure response in the bottom of a downcomer tray column to vapour flow rate disturbance was considered to be immediate, without delay, provided liquid holdup was independent of vapour rate. Here a simplified mathematical model is developed for the case of shower tray columns, where liquid holdup depends strongly on vapour rate. According to this model, the column pressure response for both vapour and liquid rate disturbances is the same and for moderate tray numbers up to 15 it can be approximated by the response of a first-order element with both the gain and the time constant equalling n times those of a single tray.

The column pressure transient responses and frequency functions are computed up to 100 trays. Resonance effects appear in the computed frequency functions above 15 trays: at moderate frequencies there is a periodic fluctuation of both the gain and the phase lag. This is the explanation of the phenomenon found earlier experimentally in an industrial column [4].

An interpretation of the resonance effect is given by the existence of the higher order trays as side capacities. For low frequencies the computed responses show the same type of first-order behaviour as columns with small tray numbers. For increasing frequencies the response of the whole column tends to that of a single tray.

Summary

A simplified mathematical model is developed for the pressure response to vapour and liquid flow rate disturbances in the bottom of a distillation (or absorption) column in the case of trays without downcomers. Transient responses to step disturbance and frequency functions have been computed for columns with tray numbers up to 100. The responses for small tray numbers, or for more than 15 trays but low frequencies can be approximated by first-order behaviour. In the case of high tray numbers resonance effects appear in the frequency response.

Notation

A, B		constants, [mm of water]
a, b		constants, [mm of water/l], [mm of water/(l/s)]
G		symbol of transfer function
H		liquid holdup, [l]
j		$\sqrt{-1}$
K		gain factor of a tray, [mm of water/(l/s)]
L		liquid flow rate, [l/s]
n		total number of trays in the column
p		pressure, [mm of water]
s		Laplace transform variable
T		tray time constant, [s]
τ		column time constant, [s]
t		time, [s]
V		vapour flow rate, [m^3/s]
Δp		pressure drop, [mm of water]
Θ		temperature, [K]
ω		frequency of sinusoidal forcing, [1/s]
Subscripts	B	reboiler
	i	tray number
	S	heating steam
Superscript	Δ	deviation from steady state

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