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MASS TRANSFER AT MIXING PROCESSES

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Abstract

In this paper the oxygen transfer in a mixing process is analyzed and measured. The tests were performed in a glass vessel with 30 l water content and a loop mixer was used as a mixing equipment. The results proved a good agreement with $K_l \cdot a$ values calculated by relationships proposed by the authors of [8] and [9], but the values calculated on the basis of two other papers show significant differences.

Moreover, we have found out that the procedure recommended for $K_l \cdot a$ estimation by the author of [5] is not quite correct, because it does not consider the changes in the oxygen electrode time delay.

A new non-dimensional relationship was proposed for $K_l \cdot a$ calculation and the results obtained showed good agreement between theory and practice.

Keywords: overall volume coefficient of oxygen transfer, oxygen concentration, loop mixer equipment, oxygen electrode, oximeter, dynamometer.

1. Introduction

Homogenization of various mixtures by a mixing process is often connected with mass transfer. Usually it is oxygen transfer to the continuous (liquid) phase – in a two-phase flow – which can be accompanied by a transfer into the solid phase. It is a situation which occurs in an aerobic bioreactor. Here the mixer serves not only for intensive content homogenization by supporting momentum transfer, but also for higher mass transfer expressed by the overall volume mass transfer coefficient $K_l \cdot a$. This parameter is one of the important indicators characterizing the quality of bioreactor operation.

The fact that the mixing process supports not only bubble distribution, but directly influences also the magnitude of the coefficient $K_l \cdot a$ is evidenced by its dependence on the power used for the mixing. In Refs. [1] – [6] this dependence is given in the following form:

$$K_l \cdot a = B \cdot \left(\frac{P_G}{V_l}\right)^{\alpha} \cdot j_g^{\beta}. \tag{1}$$

But here it is necessary to mention that relationship (1) is dimensionally inhomogeneous and therefore physically incorrect. Based on tests performed by ourselves we therefore decided to propose an equation, which removes the disadvantage mentioned above. The relationship proposed by us between the non-dimensional parameters is as follows:

$$Sh_l \cdot a \cdot d_m = C \cdot \mathrm{Re}_m^{\alpha} \cdot N^{\beta}, \tag{2}$$

where criterion N equals:

$$N = \frac{Q \cdot d_m}{v_l \cdot D^2}. \tag{3}$$

Eqs. (1) and (3) are valid for the turbulent region.

2. Methods for Determining the Coefficient $K_l \cdot a$

These methods are excellently described in Refs. [1] and [8]. Basically the coefficient $K_l \cdot a$ is determined by following the changes in the oxygen concentrations C_l in the liquid. One of the principles used is the degassing method. The method utilizes the monitoring of dissolved oxygen concentrations using an oxygen electrode suddenly transferred from a solution with $C_l = 0$ (zero solution) into test solution or a solution saturated with oxygen. The zero solution is obtained purging the solution with nitrogen. This unsteady process can be described by the following differential equation:

$$\frac{\mathrm{d}C_l}{\mathrm{d}t} = \frac{K_l \cdot a}{1-\varepsilon} \cdot (C_l^* - C_l), \tag{4}$$

which, after integration, gives a linear equation, from which $K_l \cdot a$ values can be determined:

$$\ln(C_l^* - C_l) = \ln C_l^* - \frac{K_l \cdot a \cdot t}{1-\varepsilon}. \tag{5}$$

Eq. (1) can be used when the oxygen electrode gives a quick response. The rate of response is expressed by the time delay t_d also called response time needed by the electrode to reach 90% of the total signal change during the transfer of the electrode mentioned earlier. WTW oxygen electrodes (product of FRG) (see Ref. [7]) seem to be suitable for such tests. The WTW manufacturer declared that t_d for the electrode used by us is less than or equal to 10 s. In this case a time constant t_E If illustrated in *Fig*.2 is 4s. t_E is less than 3 s, then *Eq*. (4) can be used for a direct evaluation of $K_l \cdot a$. (see Ref. [11]). In the opposite case it is necessary to consider the electrode time delay (see Refs. [5] and [8]). The author of [5] recommends, besides *Eq.* (4), to take into account also a second differential equation as follows:

$$\frac{\mathrm{d}C_E}{\mathrm{d}t} = \frac{C_l - C_E}{t_E}. \tag{6}$$

The common solution of *Eqs.* (4) and (6) gives the time dependence of the oxygen electrode:

$$C_E = C_l^* \cdot \left(1 - \frac{e^{\frac{-K_l \cdot a \cdot t}{1-\varepsilon}}}{1 - \frac{K_l \cdot a}{1-\varepsilon} \cdot t_E} + \frac{\frac{K_l \cdot a}{1-\varepsilon} \cdot e^{-\frac{t}{t_E}}}{\frac{1}{t_E} - \frac{K_l \cdot a}{1-\varepsilon}} \right) . \tag{7}$$

3. Experimental

Our tests were performed by a loop mixer equipment illustrated in *Fig.1*. It consists of a mixer element with three flat blades inclined at an angle of about 45° to the horizontal plane according to previous CSN 691025). Around the mixer element a circulation tube is situated. The tube ensures an ordered motion of the mixed content and there is no need for baffles. The mixer shaft is located in two bearings; the lower one is of sliding construction and made of graphite teflon, the upper one is a swivel one row bearing. The driving unit is a direct current electric motor with safety voltage of 48 V. Connection between the motor and the mixer shaft is realized through a Hardy disk coupling made of silone. The maximum power of the electric motor is 80 W and the declared maximum revolution frequency 1650 min^{-1}.

The mixer equipment was manufactured at Bratislavske Potravinarske Strojarne (Food Machinery Factory Bratislava SR) according to our design. Its individual parts were made of stainless steel class 17248. Part of the equipment driving unit consists of an electric current rectifier with transformer from 220 V to 48 V. In our experimental plant there was also an autotransformer enabling to change the revolution frequencies in the range from zero to the maximum value. The mixer equipment was located in a glass vessel with a diameter of 300 mm with hemispheric bottom.

The mixer revolution frequencies and shaft torsion moments were measured by a Steiger-Mohilo dynamometer (FRG product) with a resolution of 0.01 Nm. The dynamometer torsion element was located between the two Hardy coupling parts, see *Fig. 1*.

The oxygen concentrations were measured by a polarographic electrode EO 96, the signals amplified and displayed by oximeter OXI 527 (both devices products of WTW Co. FRG, see Ref. [7]). This company is well known for its products for measurement of dissolved oxygen concentration in water. The electrode was declared to have a time delay less than 10 s. Despite this fact first our measurements

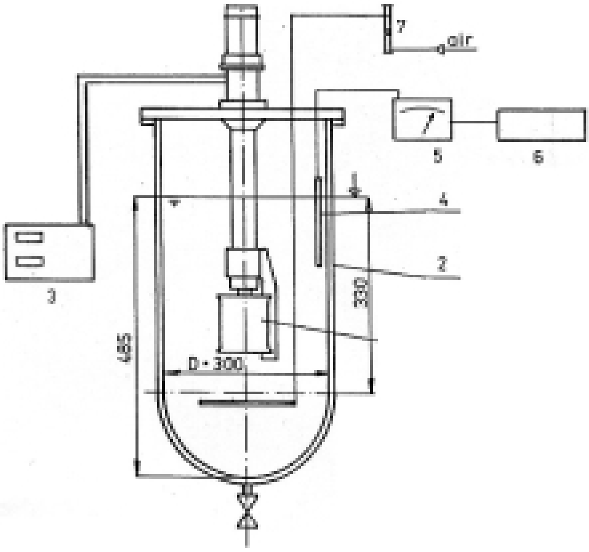

Fig. 1.

were started with an electrode head having a delay more than 20 s and only after the heard had been replaced were we able to reach the declared 10 s delay.

This fact was taken into account at $K_l \cdot a$ evaluations and despite our efforts to obtain the manufacturer's explanation of such a deviation, we did not receive any answer. In our opinion the difference was caused by the head membrane thickness. In our measurements we used a degassing method – the dissolved oxygen was replaced by nitrogen. After reaching zero concentration we overstrained a three way valve on to aeration and the recorder (double line device TZ 500, producer Verkon Co. Prague) recorded a response to this change. Each measurement was stopped after reaching 90% saturation. But in $K_l \cdot a$ calculations we did not proceed according to (7). The reason for this was that such attitude asks for an iterative calculation. From the *Fig. 2* one can see that for receiving e.g. 55% of the saturation 3 s time delay was recorded, for 77% 5 s, etc. Therefore we have proposed the following procedure:

the recorder shows, that e.g. for the 55% saturation 60 s were needed. Then from this duration the time delay 3 s was subtracted, etc.

Then relationship (5) was changed into a difference equation and $K_l \cdot a$ values were determined. During these calculations we neglected $(1-\varepsilon)$ in the denominator because the ε values were lower than 1%.

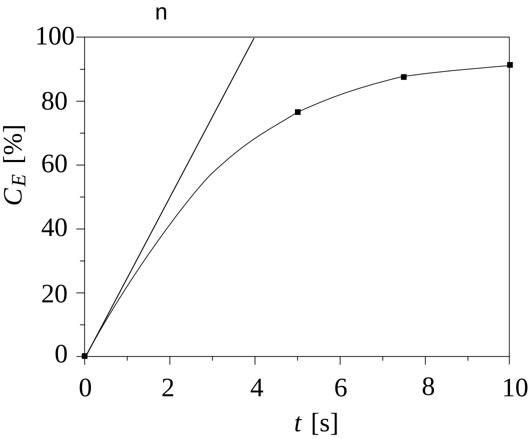

Fig. 2. Course of C_E during the sudden electrode movement from a medium with $C_1 = 0\%$ into a medium with $C_1 = 100\%$

4. Results

The vessel content was tap water, revolution frequencies changed within the range of 10–34 s^{-1}, as parameters the air flows were 0.13 l/s, 0.32 l/s, 0.40 l/s, 0.58 l/s.

The results of our measurements were obtained from line slopes and are illustrated in *Figs*. 3 and 4. From *Fig*. 4 it was possible to determine the following parameters for relationship (3):

$$C = 0.913, \qquad \alpha = 0.8, \qquad \beta = 0.25.$$

5. Discussion

A comparison of our results with other ones led to the following statements:

- our results are in good agreement with values calculated by the authors of [8] using the relationship:

$$K_l \cdot a = 0.026 \cdot \left(\frac{P_g}{V_l}\right)^{0.4} \cdot j_g^{0.5}, \tag{8}$$

and with the values calculated by the authors of (10) using the relationship:

$$K_l \cdot a = \left(\frac{P_g}{V_l}\right)^{0.475} \cdot j_g^{0.4}, \tag{9}$$

(see *Table* 1).

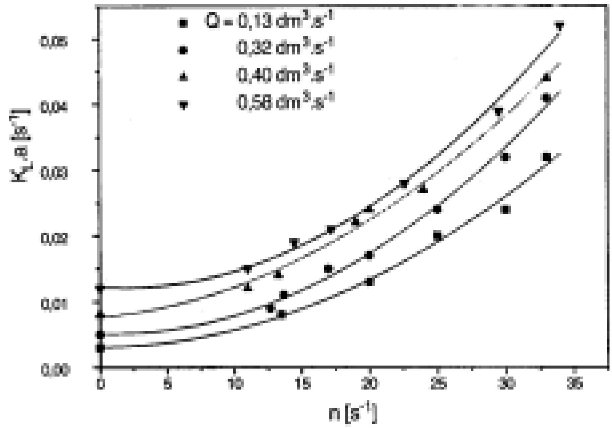

Fig. 3. $K_l \cdot a$ vs. n graph. Parameter is air flow.

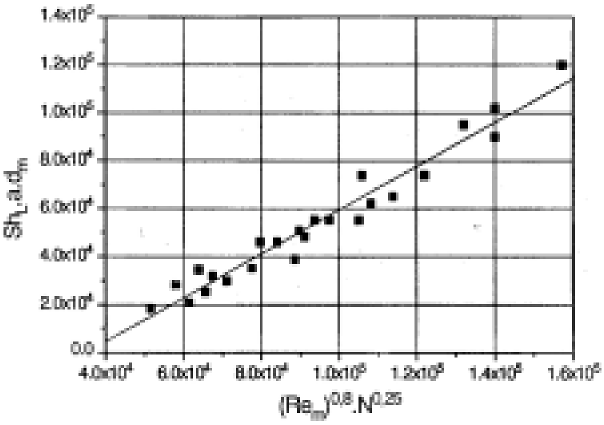

Fig. 4. Relationship between the non-dimensional parameters

- taking into account relationship given by JUDAT in [10]:

$$(K_l \cdot a)^* = 9.8 \cdot 10^{-5} \cdot \frac{\left(\frac{P_g}{V_l}\right)^{*0.4}}{B^{-0.6} + 0.81 \cdot 10^{\frac{-0.65}{B}}}, \tag{10}$$

where

$$(K_l \cdot a)^* = K_l \cdot a \cdot \sqrt[3]{\frac{v}{g^2}}, \tag{11}$$

$$\left(\frac{P_g}{V_l}\right)^* = \frac{\frac{P_g}{V_l}}{\rho_l \sqrt[3]{\upsilon \cdot g^4}}, \tag{12}$$

$$B = \frac{Q}{D^2} \cdot \frac{1}{\sqrt[3]{\upsilon \cdot g}}, \tag{13}$$

it can be stated that at low n there is a good agreement, while at high n the loop mixer yields $K_l \cdot a$ values higher by about 50%. *Eqs.* (10) to (13) satisfy the condition for dimensional homogeneity, but they are too complicated.

- and considering [3] is including the relationship

$$K_l \cdot a = 0.004 \cdot \left(\frac{P}{V_l}\right)^{0.67} \cdot j_g^{0.31}, \tag{14}$$

where P is the total power input, we must state, that it gives values, which are approximately two times higher.

Moreover, from *Fig.* 3 it is visible that for $P_g(n) = 0$ the coefficient $K_l \cdot a \neq 0$, because in this case the system acts as a bubble column and the corresponding relationship for $K_l \cdot a$ estimation is, according to [8]:

$$K_l \cdot a = 0.32 \cdot j_g^{0.7}, \tag{15}$$

and according to [11]:

$$K_l \cdot a = 0.39 \cdot j_g^{0.67}, \tag{16}$$

see *Fig.* 5.

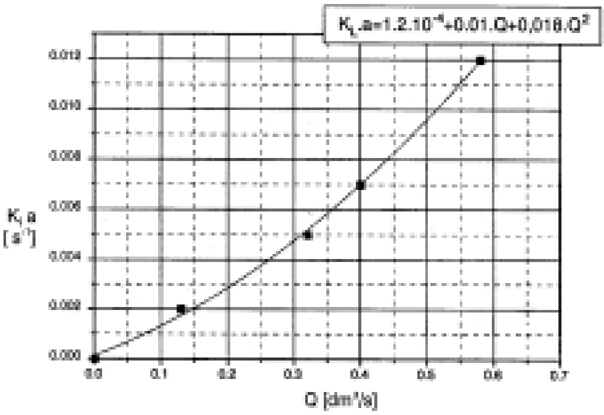

Fig. 5. Results for the case where the system acts as a bubble column

Table 1. Comparison of results received by us with the results calculated according to relationships (7) and (9)

$Q = 0.13$ dm^3/s

$K_l \cdot a$ [s^{-1}]	$P_g = 4.7$ W	$P_g = 17.6$ W	$P_g = 35.3$ W	$P_g = 54.6$ W	$P_g = 58$ W
This work	0.008	0.013	0.02	0.024	0.032
Accord. to [8]	0.008	0.014	0.019	0.022	0.023
Accord. to [9]	0.009	0.017	0.023	0.029	0.03

$Q = 0.32$ dm^3/s

$K_l \cdot a$ [s^{-1}]	$P_g = 3.2$ W	$P_g = 3.9$ W	$P_g = 7$ W	$P_g = 14.5$ W	$P_g = 20.4$ W	$P_g = 35.8$ W	$P_g = 45.6$ W
This work	0.009	0.011	0.015	0.017	0.024	0.032	0.041
Ref. [8]	0.011	0.012	0.016	0.021	0.024	0.03	0.033
Ref. [9]	0.011	0.012	0.015	0.022	0.026	0.034	0.038

$Q = 0.40$ dm^3/s

$K_l \cdot a$ [s^{-1}]	$P_g = 3.5$ W	$P_g = 5$ W	$P_g = 9.5$ W	$P_g = 10.1$ W	$P_g = 18.1$ W	$P_g = 45.6$ W
This work	0.012	0.014	0.022	0.024	0.027	0.044
Ref. [8]	0.013	0.015	0.02	0.02	0.025	0.037
Ref. [9]	0.012	0.014	0.019	0.02	0.026	0.041

$Q = 0.58$ dm^3/s

$K_l \cdot a$ [s^{-1}]	$P_g = 2.6$ W	$P_g = 4.6$ W	$P_g = 7.6$ W	$P_g = 12.8$ W	$P_g = 25.9$ W	$P_g = 40.6$ W
This work	0.015	0.019	0.021	0.028	0.039	0.052
Ref. [8]	0.014	0.018	0.022	0.027	0.035	0.042
Ref. [9]	0.012	0.016	0.02	0.026	0.036	0.045

According to *Fig.* 5

$$K_l \cdot a = 1.2 \cdot 10^{-4} + 0.01 \cdot Q + 0.018 \cdot Q^2. \tag{17}$$

A comparison between values obtained in this work and values obtained from [8] is presented in *Fig.* 6.

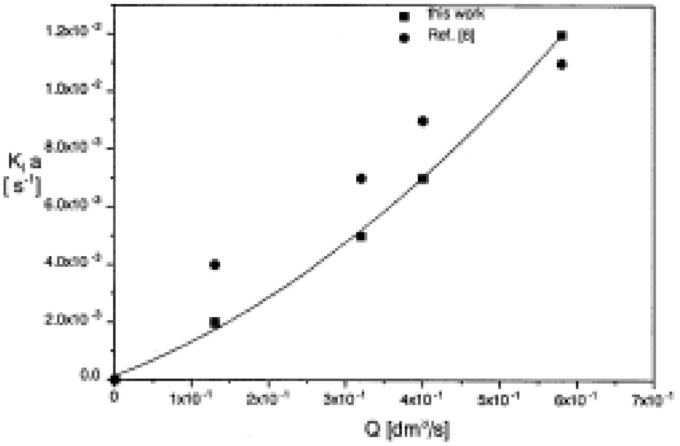

Fig. 6. Comparison between our results and results calculated according to *Eq.* (15) from Ref. [8] for $n = 0$

6. Conclusions

The performed tests proved the ability of the loop mixer equipment to provide good transport properties. Besides intensive mixing efficiency and appropriate Po number [12], the equipment is suitable also for technological processes, where content aeration is demanded. According to our design a similar loop mixer for 3000 l vessel content was manufactured in BPS Bratislava and was successfully used for a thick juice clarification process at Dunajska Streda Sugar Factory.

Our tests have pointed to the following facts:

- In the literature several summarizing relationships have been introduced for $K_l \cdot a$ determination in classical mixing processes but the results obtained from them differ by nearly about 100%,
- The values obtained by us were in good agreement with the results calculated using the relationships in [8] and [9]. But it necessary to stress also here that for $n = 0$ ($P_g = 0$) the value of $K_l \cdot a \neq 0$, because in this case the equipment acts as a bubble column.

- The electrode time delay is not constant; therefore we have recommended a procedure for taking this fact into consideration in calculating $K_l \cdot a$.
- The relationship for $K_l \cdot a$ determination can be written in the following non-dimensional form:

$$Sh_l \cdot a \cdot d_m = 0.913 \cdot \mathrm{Re}_m^{0.8} \cdot N^{0.25} \tag{18}$$

- $K_l \cdot a$ values at high n show that the testing mixer equipment is suitable for oxygen transfer intensification (see the results in the *Table* 1).

Our future work will be directed on tests with different vessel contents – e.q. saccharose – water solutions, CMC in water solutions in order to precisely determine the parameters in relationship (3). Moreover, we plan to study also three=phase systems, where the solid phase will be provided by glass spheres with 0.3 mm diameter.

7. Symbols

B, C	constants	
C_l, C_l^*, C_E	oxygen concentration – actual, saturated, electrode	[%]
d_m	diameter of mixer	[m]
D	diameter of vessel	[m]
D_l	diffusivity	[m^2/s]
g	gravitational acceleration	[m/s^2]
j_g	air flux	[m/s]
$K_l \cdot a$	overall oxygen transfer coefficient	[1/s]
n	revolution frequency	[1/s]
P_g	power for aerated content	[W]
Q	air flow rate	[1/s]
t, t_d, t_E	time, electrode time delay, electrode time constant	[s]
V_l	content volume	[m]

Greek letters

α, β	exponents	[–]
ε	air hold-up	[–]
υ_l	liquid kinematic viscosity	[m^2/s]
ρ_l	liquid density	[kg/m^3]

Non-dimensional criteria:

$Sh_l = \dfrac{k_l \cdot d_m}{D_l}$ Sherwood number

$\mathrm{Re}_m = \dfrac{n \cdot d_m^2}{\upsilon_l}$ modified Reynolds number

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