A NEW APPROACH TO THE CALCULATION OF THE VARIATION OF THE MIXING LENGTH OVER THE PIPE DIAMETER

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Abstract

Despite the approximate nature of the Prandtl concept of the mixing length, it remains one of the most easy and useful method in the prediction of the velocity distribution. It had been successfully used in the prediction of the velocity distribution in many practical problems (i. e. in pneumatic conveying). Since Prandtl's assumption on the mixing length $L/r = 0{,}14 - 0{,}08(1-y/r)^2 - 0{,}06(1-y/r)^4$ is not suitable for Re number lower than 10^5, a new approach is developed, which is also valid for Re lower than 10^5, for both smooth and rough pipes.

Introduction

Prandtl has presented an empirical Eq. (1), for calculating the mixing length as a function of position along the pipe diameter. This equation is applicable for both smooth and rough pipes, but it is only valid for $Re > 10^5$. The constants of this equation are independent of the Re number.

$$\frac{L}{r} = 0{,}14 - 0{,}08\left(1-\frac{y}{r}\right)^2 - 0{,}06\left(1-\frac{y}{r}\right)^4 \tag{1}$$

Equation (1) shows the two Prandtl's hypotheses, the first being that the mixing length at the wall is zero, and the second, that $L = \alpha y$ is confirmed for small distances from the wall. With $\alpha = 0{,}4$ it can shown that

$$\left(\frac{dL}{dy}\right)_{y=0} = 0{,}4$$

The purpose of this paper is to extend the validity of Eq. (1) for $Re < 10^5$ and to increase the accuracy.

Results and discussion

Our work is based on the experimental work of Nikuradse [1, 2]. Who made an extensive experimental work on measuring the mixing length over the pipe diameter for the smooth pipe [1] and the velocity gradient over the pipe diameter for rough pipe. The measurements were based on a very wide range of Reynolds numbers $4 \cdot 10^3 \leq Re \leq 3240 \cdot 10^3$.

For smooth pipes

By plotting the $\frac{L}{r}$ value vs $\frac{y}{r}$ for every Reynolds number (Fig. 1.a, 1.b), it is shown that the value of the mixing length depends also on the Re number, especially for Re number ranges below 10^5. From these experimental results [1] and by an Eq. (2) having the same shape as Eq. (1), and by taking into

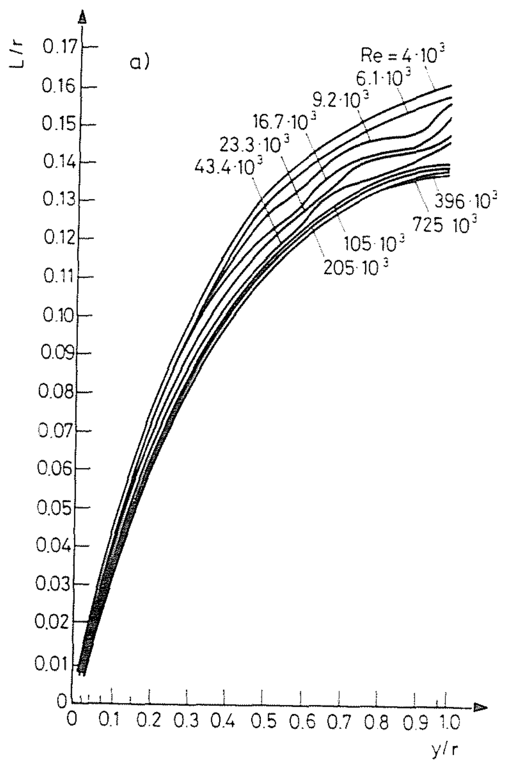

Fig. 1.a. Variation of the measured mixing length [1] over pipe diameter for smooth pipes at different Re numbers

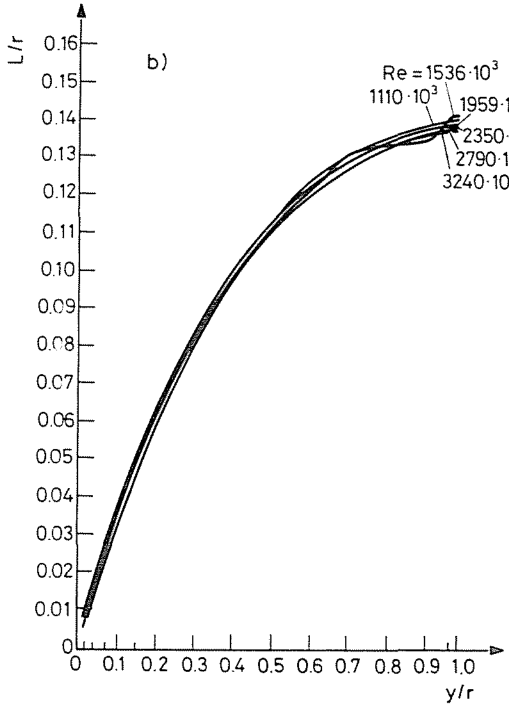

Fig. 1.b. Variation of the measured mixing length over pipe diameter for smooth pipe at different Re numbers

consideration the Prandtl's first hypothesis (3), the C value is found by the least square method for every Re number (Table 1)

$$\frac{L}{r}=C-A\left(1-\frac{y}{r}\right)^2-B\left(1-\frac{y}{r}\right)^4 \tag{2}$$

$$C=A+B \tag{3}$$

From Prandtl's second hypothesis $\left(\frac{dL}{dy}\right)_{y=0}=0,4$ and equation (2)

$$2A+4B=0.4 \tag{4}$$

From Eqs (3) and (4) it follows that

$$A=2C-0.2 \tag{5}$$

$$B=-C+0.2 \tag{6}$$

Table 1

For smooth pipe

	$Re \cdot 10^{-3}$	C
1	4	0.15817
2	6.1	0.15517
3	9.2	0.15147
4	16.7	0.14744
5	23.3	0.14496
6	43.4	0.14259
7	105	0.13997
8	205	0.13878
9	396	0.13792
10	725	0.13685
11	1110	0.13913
12	1536	0.13684
13	1959	0.13760
14	2350	0.13798
15	2790	0.13774
16	3240	0.13847

In calculating the C value by the least square method, Prandtl's second hypothesis (4) has been taken into consideration. This leads to Eq. (2) with only one constant, and by comparing the mixing length calculated by this C value with the experimental result for different Re numbers [1], it did not seem to be a good result. That is the reason why we took into consideration Prandtl's second hypothesis after the least square method. The values of C vs Re numbers (Table 1) are plotted in Fig. 2. It is clearly shown that an

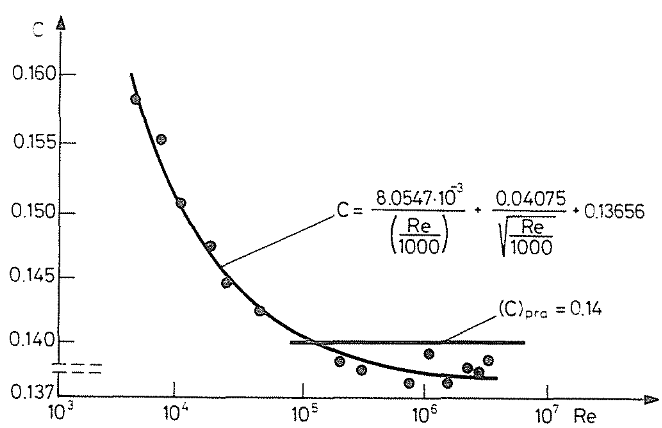

Fig. 2. The relation between C and Re for smooth pipes

empirical relation exists between C and Re values. Thus it was attempted to derive an equation for calculating the C value for any Re number:

$$C=\frac{K_1}{\left(\frac{Re}{1000}\right)}+\frac{K_2}{\sqrt{\left(\frac{Re}{1000}\right)}}+K_3 \tag{7}$$

Constants K_1, K_2 and K_3 have been determined by the least square method. So Eq. (7) becomes

$$C=\frac{8.0547\cdot 10^{-3}}{\left(\frac{Re}{1000}\right)}+\frac{0.0407}{\sqrt{\left(\frac{Re}{1000}\right)}}+0.1365 \tag{8}$$

Comparing Prandtl's equation (1), where $C=0.14$, and the developed approximation method Eqs (2, 5, 6, 8) with the experimental results [1] shows that this approach is not only valid for Re numbers below 10^5, but it also gives a better overall agreement with the experimental data than Eq. (1) (see Table 2 and Fig. 2).

Table 2

$Re\cdot 10^{-3}$	$\sigma\cdot 10^3$	
	developed approach (2, 5, 6, 8)	equation (1)
4	6.565	–
6.1	6.106	–
9.2	5.916	–
16.1	5.422	–
23.3	4.138	–
43.4	3.259	–
105	2.545	2.555
205	2.350	2.442
396	1.733	2.161
725	1.961	2.792
1110	1.388	1.548
1536	2.225	3.233
1959	1.619	2.541
2350	1.603	2.451
2790	1.955	2.847
3240	2.113	2.744

Rough Pipes

Using the velocity gradient in rough pipes measured by Nikuradse [2], for relative roughnesses of 507; 252; 126; 30.6 and 15, the mixing length along the diameter has been calculated for different Re numbers by Eq. (9)

$$L = \frac{\overset{*}{V} \cdot \sqrt{1 - \frac{y}{R}}}{\left(\frac{dv}{dy}\right)} \tag{9}$$

The C values for different relative roughnesses and different Re numbers are calculated by Eq. (2) and by the least square method, and are presented in Table 3.

Comparing Eq. (8) with the above calculated results for different relative roughnesses and Re numbers, it can be seen that Eq. (8) gives in this case a better agreement with experimental results than Eq. (1). (See Table 4 and Fig. 3.)

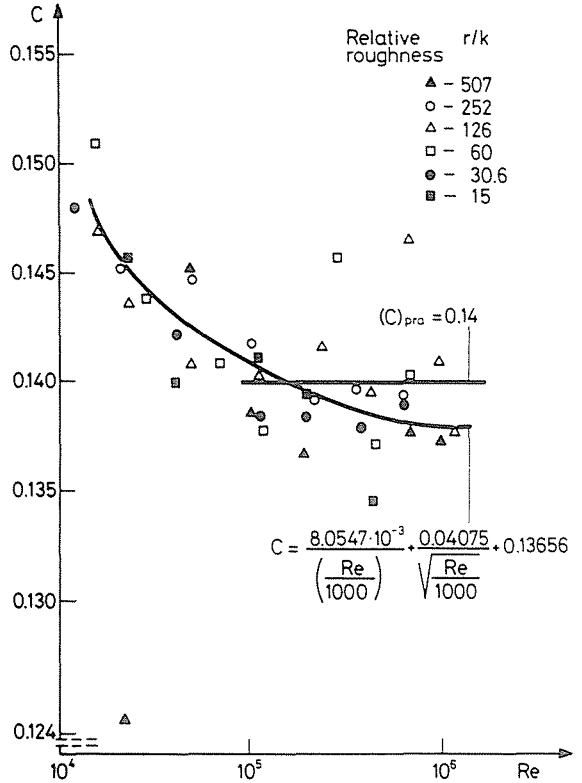

Fig. 3. The relation between C and Re for rough pipes

Table 3
For rough pipe

$Re \cdot 10^{-3}$	C $r/K = 507$	$Re \cdot 10^{-3}$	C $r/K = 252$	$Re \cdot 10^{-3}$	C $r/K = 126$	$Re \cdot 10^{-3}$	C $r/K = 60$	$Re \cdot 10^{-3}$	C $r/K = 30.6$	$Re \cdot 10^{-3}$	C $r/K = 15$
22.7	0.1246	21.4	0.1452	16.7	0.1469	15.3	0.1509	12.1	0.148	11.3	0.1238
49	0.1450	51	0.1446	23.7	0.1435	29.5	0.1438	23	0.1706	22.2	0.1456
106	0.1385	103.5	0.1416	50.5	0.1406	70	0.1409	43	0.1420	43	0.1399
186	0.1367	202	0.1390	112	0.1402	116	0.1378	104	0.1384	108	0.1410
427	0.1343	344	0.1394	231	0.1415	271	0.1455	195	0.1383	197	0.1394
680	0.1377	624	0.1392	417	0.1394	438	0.1369	372	0.1379	430	0.1345
970	0.1373			640	0.1463	677	0.1401	638	0.1387		
				960	0.1407						

Table 4

R/K	$\sigma_{abs(max)}$
507	$5.0068 \cdot 10^{-3}$
252	$4.505 \cdot 10^{-3}$
126	$5.094 \cdot 10^{-3}$
60	$6.413 \cdot 10^{-3}$
30.6	$5.544 \cdot 10^{-3}$
15	0.07468

Nomenclature

L	= mixing length
Re	= Reynolds number
V	= velocity in axial direction
y	= distance normal to pipe
r	= radius of the pipe axis
$\overset{*}{V}$	= shear-stress velocity
C, A, B	= constants in equation (2)
K_1, K_2, K_3	= constant in equation (7)
r/K	= relative roughness
σ	= standard deviation

References

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2. NIKURADSE, J.: Strömungsgesetze in rauhen Rohren. Forsch., Arb. Ing-Wes. No. 361 (1933).

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