PARTICLE VELOCITY AFTER COLLISION

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Abstract

An experimental method has been developed to determine the values of the two components of the particle velocity after collision as a function of impact angle and velocity. The experimental work was performed on four types of spherical particles (aluminium oxide, glass, polystyrene and fertilizer) impacted on steel plate of 1 mm thickness, in the impact angle range of $11° < \alpha_e < 90°$. Linear relation was found between the difference in normal components of the particle velocity before and after collision and the sine of the impact angle. The extension of this relation to the small impact angle range provided the same results as those found by Muschelknautz, the value of the normal component of the particle velocity after collision is higher than that before collision, which means that at small impact angles the normal coefficient of collision is higher than 1, thereby the continuous movement of the particles in a horizontal pneumatic conveying system can be explained. It was found that at large impact angles the change in the normal component of the particle velocity before and after collision is greater in the case of large particles. Generally the change in the parallel components of the particle velocity before and after collision is greater in the case of small particles.

Due to the simplicity of the experimental work, this method can be easily used in the prediction of particle velocity after collision.

Introduction

To our knowledge there is no numerical method to calculate the particle velocity after collision without experimental work. The experimental work was carried out to find the value of the restitution and friction coefficients. Various definitions are available for the restitution coefficient, BRAUER [1] and SHELDON [2] used the following:

$$\text{Restitution coefficient} = CP_u/CP_e\,, \tag{1}$$

where CP_u and CP_e are the particle velocity after and before collision, respectively.

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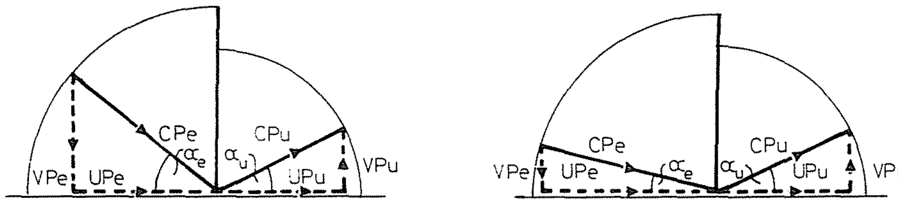

Fig. 1. Collision of the particle with rigid surface

OKUDA [3], OTTJES [4], SAWATZKI [5], TABOR [6], YAMAMOTO [7] and TSUJI [8] used:

$$\text{Restitution coefficient} = VP_u/VP_e \qquad (2)$$

where VP_u and VP_e are the normal components of the particle velocity after and before collision, respectively.

To be able to make distinction between the above ratios, the following definitions will be used in this work:

coefficient of collision $e_0 = CP_u/CP_e$
normal coefficient of collision $e_n = VP_u/VP_e$
tangential coefficient of collision $e_t = UP_u/UP_e$
restitution coefficient $e = (CP_u/CP_e)_{\alpha_e} = 90°$

In the case of normal impact $e_n = e = e_0$, (Fig. 1).

	Angle	
	large angle	small angle
$e_0 = CP_u/CP_e$	: <1	: $e_0 < 1$
$e_n = VP_u/VP_e$	: <1	: $e_n > 1$
$e_t = UP_u/UP_e$	: <1	: $e_t < 1$
$e = (CP_u/CP_e)_{\alpha_e} = 90°$	: <1	: $e < 1$

According to OKUDA [3], TABOR [6] and SAWATZKI [5] the normal coefficient of collision depends on the normal component of the particle impact velocity. YAMAMOTO [7] and BRAUER [1] established that the normal coefficient of collision depends on the impact angle. Because of the difficulty in measuring the e_n value at small impact angles, it was assumed by BRAUER [1] that at an impact angle $\alpha_e = 0°$ the $e_n = 1$ and the curve is extended to the nearest measured value. SALMAN [9] reported that the restitution coefficient e depends on the particle diameter and on the target thickness, too. Although it is known that the effect of the wall thickness on the value of e can be neglected if the wall thickness is greater than a certain value. In this examination a steel target of 1 mm thickness has been used, because the same was used as wall thickness

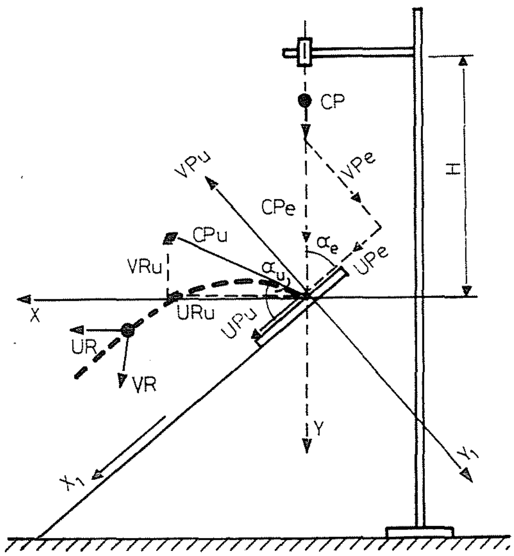

Fig. 2. Particle velocity before and after collision

of a rectanglular duct in the experimental investigation of a pneumatic conveying system.

The aim of this work is to present an experimental method to determine the two components of the particle velocity after collision, as a function of the impact angle and particle velocity before collision, and to show that at small impact angles the normal component of the particle velocity after collision is higher than that before collision, that is at small impact angles the value of the normal coefficient of collision e_n is higher than 1.

Experimental work

The experimental work has been carried out by dropping particles from a certain distance *H* and recording their trajectory after collision with a steel plate of 1 mm thickness, Fig. 2. The trajectory of the particle has been recorded by a video tape recorder. The same experiment was performed for different impact angles and for four types of spherical particles of different diameters.

Particle velocity before collision

A particle released from rest will fall freely with constant acceleration. By neglecting the drag force, the impact velocity of the particle can be calculated by

$$CP_e = \sqrt{2Hg} \tag{3}$$

The differential equation of motion (4), with the initial values: $t=0$ and $Y=-H$ has been used to check the result of Eq. (3).

$$\frac{dCP}{dt} = g - \frac{3}{4d}\frac{\varphi_a}{\varphi_P} CD\, CP^2 \tag{4}$$

CD is the drag coefficient expressed by the Kaskas equation:

$$CD = \frac{24}{R_{eP}} + \frac{4}{\sqrt{R_{eP}}} + 0.4: \qquad R_{eP} = \frac{CPd}{\vartheta}$$

In our case the relative error was found to be less than 1.8%.

The change of the impact angle does not influence the particle velocity before collision CP_e, it affects only the value of its components.

Particle velocity after collision

UR_u and VR_u are the components of the particle velocity CP_u after collision in the X, Y coordinate system and they are calculated from the particle trajectory. By neglecting the drag force, the only force exerted on the particle is its weight, the horizontal component of acceleration is zero and the vertical component is equal to that of a freely falling body.

Since the horizontal acceleration is zero, the horizontal component of velocity UR remains constant, so $UR = UR_u$.

The X coordinate of the particle trajectory after collision is at any time equal to

$$X = UR_u \cdot t \tag{5}$$

The Y coordinate at any time is

$$Y = VR_u \cdot t - \frac{1}{2} g \cdot t^2 \tag{6}$$

from Eqs (5 and 6) it follows that

$$Y = X\left(\frac{VR_u}{UR_u}\right) - X^2\left(\frac{g}{2\,UR_u^2}\right) \tag{7}$$

At least two points of the particle trajectory are needed to calculate UR_u and VR_u. To achieve more exact results, 12 points of the trajectory have been used, and UR_u and VR_u have been calculated by the least square method, for particle trajectories at different impact angles. CR_u is the particle velocity after collision in the coordinate system X, Y.

$$CR_u = \sqrt{UR_u^2 + VR_u^2} \tag{8}$$

The components of the particle velocity parallel with and normal to the wall UP_u and VP_u have been determined by transforming the CR_u into the coordinate system X_1, Y_1.

Result and Discussion

1— *Normal component of the particle velocity after collision*

Figure 3 demonstrates the relation between the difference of the normal component of the particle velocity before and after collision (ΔVP) divided by the particle impact velocity before collision CP_e and the sine of the impact angle α_e for four types of spherical particles of different diameters, where $\sin \alpha_e = VP_e/CP_e$. The Figure shows that a linear relation exists between $\Delta VP/CP_e$ and $\sin \alpha_e$ in the examined angle range. The extension of this linear relation to the small impact angle range shows that there is a critical angle α_c for which the normal component of the particle velocity after collision is equal to that before collision ($VP_u = VP_e$), for POL particle $\sin \alpha_c = 0.202$ as shown in Fig. 3 and Table 1. Further extension in the direction of small angles shows that the normal component of the particle velocity after collision is larger than the component before collision ($VP_u > VP_e$). This means that at small impact angles the value of the normal coefficient of collision e_n is higher than 1, providing a good explanation to the continue flying of the particles in a horizontal pneumatic conveying system where the impact angles are always very small.

An approximate equation for the relation between $\Delta VP/CP_e$ and VP_e/CP_e is proposed in the form of Eq. (9):

$$\frac{\Delta VP}{CP_e} = (1-e) + K_1 \left(1 - \frac{|VP_e|}{CP_e}\right) \tag{9}$$

where e is the restitution coefficient. The values of e have been reported in a previous paper [9], Table 1.

The constant K_1 was determined by the least square method from the values shown in Fig. 3. The values of K_1 for different particles are presented in Table 1.

Figure 3 illustrates that in the large impact angle range, the value of $\Delta VP/CP_e$ is higher in the case of large particles than in the case of small ones.

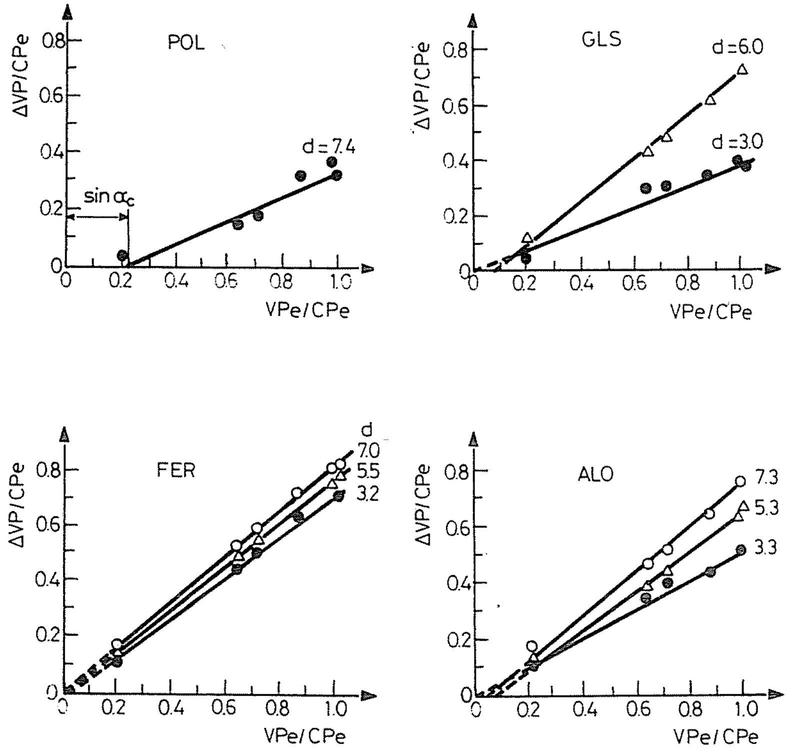

Fig. 3. The relation between $\frac{VP_e}{CP_e}$ and $\frac{\Delta VP}{CP_e}$ for four types of particles (POL, GLS, FER and ALO)

Table 1

	ALO			GLS		POL	FER		
	d=3.3	d=5.3	d=7.3	d=3.0	d=6.0	d=7.4	d=3.2	d=5.5	d=7.0
e	0.5	0.36	0.25	0.59	0.25	0.65	0.274	0.20	0.175
K_1	−0.504	−0.704	−0.790	−0.407	−0.809	−0.439	−0.746	−0.808	−0.827
$\sin\alpha_c$	0.0079	0.090	0.0506	0.0073	0.0729	0.202	0.0268	0.0099	0.00241

It is worth comparing the results above with the results presented by MUSCHELKNAUTZ [10] and by BRAUER [1]. MUSCHELKNAUTZ [10] carried out accurate measurements, he found that in the small impact angle range the normal component of the particle velocity after collision is higher than that of prior to collision. He applied a centrifugal type rig to state the relation

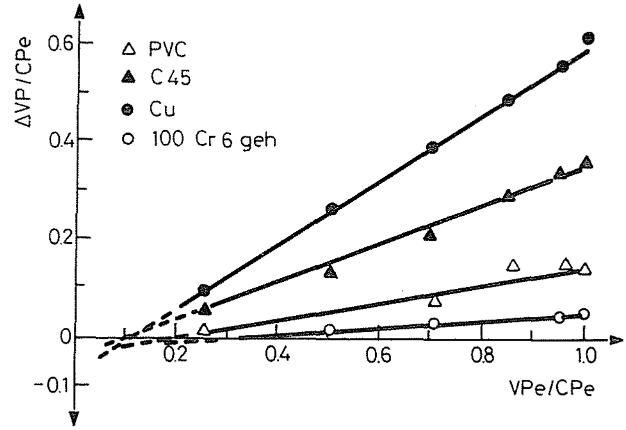

Fig. 4. The relation between $\frac{VP_e}{CP_e}$ and $\frac{\Delta VP}{CP_e}$ calculated from Brauer's measurements [1] by Eq. (10)

between $\Delta VP/CP_e$ and VP_e/CP_e for 7 types of particles and 5 types of surface materials. His experiments have been performed in the impact angle range of $5° < \alpha_e < 30°$.

BRAUER [1] carried out his measurements on steel particles of $d = 6.0$ mm impacted on 12 types of target surface. He measured the e_0 and $\Delta\alpha$ values as a function of α_e. BRAUER did not observe the linear relation between $\Delta VP/CP_e$ and $\sin \alpha_e$ because he used another coordinate system. In order to find the relation between $\Delta VP/CP_e$ and VP_e/CP_e from BRAUER's measurements, Eq. (10) has been used:

$$\frac{\Delta VP}{CP_e} = \sin \alpha_e - e_0 \sin \alpha_u \tag{10}$$

where $\sin \alpha_e = VP_e/CP_e$ and $\sin \alpha_u = VP_u/CP_u$.

For clarity the results of only four types of particle are shown in Fig. 4. This figure also shows a linear relation between $\Delta VP/CP_e$ and VP_e/CP_e in the examined angle range. The extension of this relation to the small impact angle range shows that $VP_u > VP_e$.

2— *Parallel component of the particle velocity after collision*

Figure 5 shows the relation between $\Delta UP/CP_e$ and the sine of the impact angle for four types of particles (ALO, POL, GLS and FER), where ΔUP is the difference between the components of the particle velocities parallel to the wall before and after collision ($\Delta UP = UP_e - UP_u$).

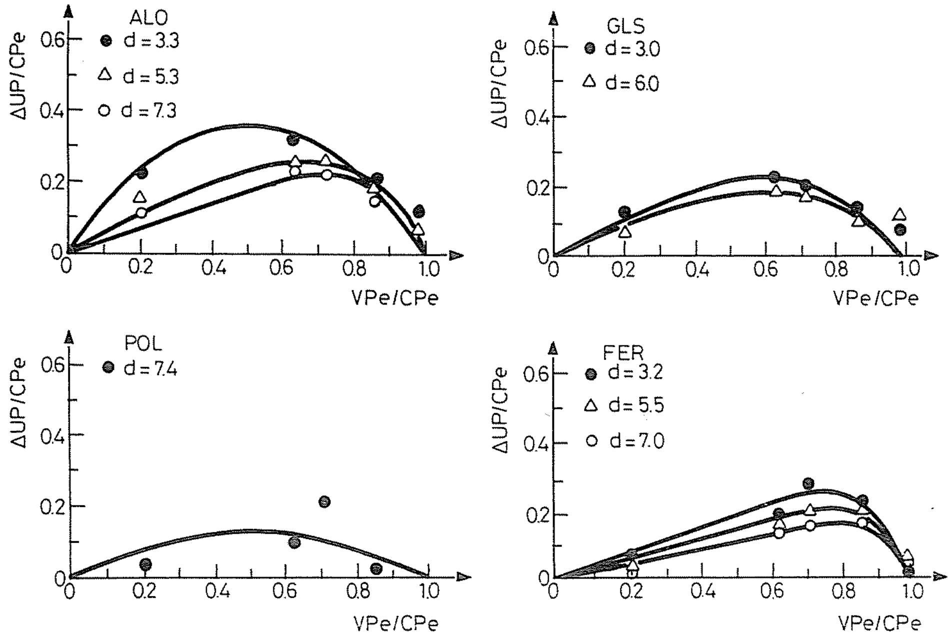

Fig. 5. The relation between $\frac{VP_e}{CP_e}$ and $\frac{\Delta UP}{CP_e}$ for four types of particles (ALO, GLS, POL and FER)

An approximate equation for the relation between $\Delta UP/CP_e$ and VP_e/CP_e is proposed in the form of Eq (11):

$$\frac{\Delta UP}{CP_e} = K_2\left(\frac{|VP_e|}{CP_e} - \left(\frac{|VP_e|}{CP_e}\right)^{K_3}\right) \tag{11}$$

The values of the constants K_2 and K_3 have been determined by the least square method. Their values are shown in Table 2.

In Fig. 5 it can be seen that to an impact angle lower than $\alpha_e = 51°$ where $VP_e/CP_e = 0.78$ the difference between the components of the particle velocity parallel to the wall before and after collision is greater in the case of small particles.

On the basis of BRAUER's experiments [1] and using Eq. (12), Fig. 6 can be drawn showing the relation between $\Delta UP/CP_e$ and VP_e/CP_e.

$$\frac{\Delta UP}{CP_e} = \cos\alpha_e - e_0 \cos\alpha_u \tag{12}$$

MUSCHELKNAUTZ [10] described a linear relation between $\Delta UP/CP_e$ and VP_e/CP_e in the angle range $(0° < \alpha_e < 17°)$.

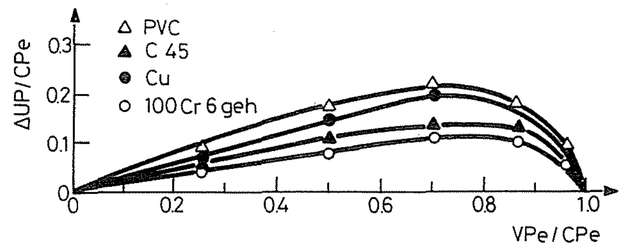

Fig. 6. The relation between $\frac{VP_e}{CP_e}$ and $\frac{\Delta UP}{CP_e}$ calculated from Brauer's measurements [1] by Eq. (12)

Table 2

	ALO			GLS		POL	FER		
	d=3.3	d=5.3	d=7.3	d=3.0	d=6.0	d=7.4	d=3.2	d=5.5	d=7.0
K_2	1.494	0.541	0.372	0.544	0.424	0.544	0.384	0.286	0.233
K_3	2.0	4.0	7.0	3.5	4.0	2.0	7.9	10.6	12.0

Particle collision in pneumatic conveying systems

In horizontal pneumatic conveying systems solid particles in the course of their way in the pipe hit the wall many times. According to the experiments carried out in this field, the particles continue their movement in the pipe if the air velocity is higher than the critical value. As the aerodynamic forces are not enough to keep relatively large particles in suspension in the pipe and the $e_n \leq 1$, as was supposed by other authors, the particles will settle down on the bottom of the pipe after a few collisions.

To solve the problem MATSUMOTO [11, 12] proposed a model based on the random and abnormal collision due to the roughness of the wall surface and/or to the irregularity of the particle shapes. TSUJI [8] elaborated another model based on that, when the impact angle becomes smaller than a certain value the wall is replaced by a virtual one with a certain angle against the true wall.

The presented model can explain the continuous movement of the particle in horizontal pneumatic conveying systems. If the particle impact angle is less than the critical angle α_c as shown in Fig. 3, the normal component of the particle velocity after collision will be higher that before collision. Thus the continuous flying of the particle will be ensured.

Nomenclature

CP : particle velocity in the coordinate system X_1, Y_1 (m/sec)
CR : particle velocity in the coordinate system X, Y (m/sec)
CD : particle drag coefficient
d : particle diameter (m)
e : restitution coefficient $(CP_u/CP_e)_{\alpha_e}=90°$
e_0 : coefficient of collision CP_u/CP_e
e_n : normal coefficient of collision VP_u/VP_e
e_t : tangential coefficient of collision UP_u/UP_e
g : acceleration of gravity (m/sec^2)
H : the height from which the particle is dropped (m)
K_1 : constant in Eq. (9)
K_2&K_3: constants in Eq. (11)
φ_a : density of air (kg/m^3)
φ_p : density of particle (kg/m^3)
Rep : particle Reynolds Number
Rv : relative error
t : time needed for the particle to travel from the collision point to point X, Y (sec)
UR : horizontal component of the particle velocity in the coordinate system X, Y (m/sec)
VR : vertical component of particle velocity in the coordinate system X, Y (m/sec)
UP : component of the particle velocity parallel to the wall (in the coordinate system X_1, Y_1) (m/sec)
VP : component of the particle velocity normal to the wall (in the coordinate system (X_1, Y_1) (m/sec)
ΔUP : $UP_e - UP_u$ (m/sec)
ΔVP : $VP_e - VP_u$ (m/sec)
α_e : impact angle (degree)
α_u : reflection angle (degree)
α_c : critical angle where $VP_e = VP_u$ (degree)
$\Delta\alpha$: $\alpha_e - \alpha_u$ (degree)
ϑ : kinematic viscosity of air (m^2/sec)

Suffix

e : value before collision
u : value after collision

Subscripts:

ALO : Aluminium oxide
GLS : Glass
POL : Polystyrene
FER : Fertilizer

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